EFFECTS OF VARIOUS GROUPING PRACTICES ON SIXTH-GRADE MATHEMATICS STUDENTS

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The purpose of this study was to examine, in terms of constructivist theory, the opportunities to construct mathematical knowledge available to sixth-grade students when a teacher's classes were variously grouped for instruction. Qualitative methods of observation and in-depth interviewing were used for the study which was conducted at one middle school during the course of an entire school year. Informants for the study included 160 sixth-grade students and their mathematics teacher. The study was primarily concerned with the quality of mathematical interactions and the differences in delivery of curriculum and instruction that occurred in the various classes.

The major finding from the study suggests that the interactions that take place between students and the mathematical environment in which they are placed may be responsible for differences in students' mathematical achievement. Grouping arrangements affected the teacher's approach to instruction and also the social and mathematical interactions that took place between students, and between students and the teacher.

Constructivist activities were more prevalent in the heterogeneous pilot class. Varying instructional practices and a challenging curriculum engaged all students in meaningful mathematical and social interactions; the pilot class became a community of mathematical learning. Both the basic and the advanced classes tended to be more traditional and teacher directed than the pilot class. Students in the basic classes had few meaningful mathematical experiences; the curriculum consisted mainly of computation and was very similar to that of previous years. Findings from the study suggest that students of all abilities benefitted when differences in mathematical ability were accomodated through instructional rather than curriculum variations.

CHAPTER 1
BACKGROUND FOR THE STUDY

During the last decade many reports have documented the general pattern of declining academic achievement in American schools (Dossey, Mullis, Lindquist, & Chambers, 1988; McKnight, Crosswhite, Dossey, Kifer, Swafford, Travers, & Cooney, 1987; National Research Council, 1989). Other major national reports, such as the NCTM's Standards (National Council of Teachers of Mathematics, 1989) and Everybody Counts (National Research Council, 1989), argued for a reconsideration of the nature of mathematics and what constitutes effective mathematics education.

Berger and Luckmann's (1967) theory of the social construction of reality supports the current study, which added to attempts to explain the continuing decline of students' academic performance. The belief that members of a social group individually, and collaboratively, construct and maintain a distinct reality has significant implications for all facets of a theory of knowledge (Richards & von Glasersfeld, 1980). When mathematics is no longer regarded as a fixed body of knowledge, implications follow for how classroom teachers should teach and evaluate students, and even for the selection of appropriate research methods (Davis, Maher, & Noddings, 1990).

The theoretical perspective that academic ability and knowledge are socially constructed influenced the current study. "Comparing mathematics to a tool is perhaps useful in seeing how mathematics is not a description of an external reality, but is, on the contrary a human construction, invented to achieve human purposes" (Confrey, 1990, p. 110). The view that mathematical knowledge is constructed, and reconstructed, by unique individuals and is contextually dependent, supports the current study. This view of mathematical knowledge being constructed implies a creative process, not a passive situation of knowledge transfer (Goldin, 1990; Richards & von Glasersfeld, 1980). Thinking in new ways about knowledge and how it can be created may expand educators' understanding of how children learn and may help determine effective pedagogical practices (Eisner, 1991).

Drawing on the symbolic interactionist perspective of G.H. Mead, constructivists argue that initiates have to learn the language, customs, and tools of their community, continually negotiating and renegotiating meaning as they construct knowledge (Davis, Maher, & Noddings, 1990). Piaget's model of adaptation and accommodation allows for the possibility of alternative solutions to problems. "This possibility of diversity is the essence of constructivism, and it is this aspect that makes Piaget's theories of such vast significance for any understanding of . . . how it is that we come to know mathematics" (Richards & von Glasersfeld, 1980, p.30).

According to constructivist theorists, meaningful learning occurs only when learners construct their own knowledge, individually and cooperatively, in a community that poses challenges while also offering support (Noddings, 1990). Mathematical knowledge results from in-context experiences and interaction with others. Social conventions and social interactions "often function as the most important determinants of whether an individual's knowledge is regarded as valid, or whether a mathematical or scientific concept to be taught has been 'correctly' learned" (Goldin, 1990, p.35). Constructivist methods were therefore particularly appropriate for an examination of alternative organizational and instructional practices, and their influence upon students' construction of knowledge and students' social and intellectual development.

The social construction of reality (i.e. the way schools organize educational environments) powerfully influences students' self-perceptions. Studies have shown a correlation between self-esteem and achievement, with academic self-concept declining steeply in early adolescence (Bloom, 1976; Dawson, 1987; Eccles, Adler, Futterman, Goff, Kaczala, Meece, & Midgley, 1983; Kifer, 1973; Rosenholtz & Rosenholtz, 1981; Simpson, 1981; Stipek & MacIver, 1989). Beane (1991) pointed out that the correlation is especially strong when self-esteem is situation specific, for example where students are grouped by ability for mathematics. Beane noted that because self-esteem is personally constructed out of interactions with the environment, care must be taken to provide a safe social environment that enhances self-esteem. The development of a person's mathematics self-concept is most critical during early adolescence and may be the most important determinant of mathematics achievement (Brassell, Petry, & Brooks, 1980; Callahan, 1971).

Statement of the Problem

This purpose of this study was to examine, in terms of constructivist theory, opportunities to construct mathematical knowledge available to sixth-grade students when a teacher's classes were variously grouped for instruction. The design of the study was responsive to the need for "descriptive studies capable of contrasting teachers' and students' behaviors and perceptions in alternative grouping arrangements" (Slavin, 1987, p.321). Findings from the study offer insights to the practitioner, and the researcher, by providing evidence to help resolve the conflict between theory and practice concerning organizational and instructional practices.

Theoretical Base

The theoretical framework for the study extends from the symbolic interactionist perspective to the constructivist view that all knowledge is actively constructed.

> Symbolic interactionism holds that human beings act towards things on the basis of the meaning that things hold for them. The attribution of meaning to objects is a continuous process which takes place in social contexts. Thus, symbolic interactionists focus on the perspectives through which people make sense of the world; the strategies people employ to achieve their ends; the different contexts and situations in which they define their goals; their group cultures in which they interact; and their subjective, as opposed to objective, careers. (Woods, 1988, p. 90)

Objectified realities of organizational and instructional practices must be understood in terms of their subjective meanings for students and teachers. Assumptions made by educators about cognitive structures often underlie these realities. For example, research in education frequently is based on the premise that cognitive categories are fixed bodies of knowledge. In effect, this premise prevents these categories from being regarded as social constructions. Therefore, if educators assume that they know what mathematical knowledge is, they are restricted in the ways that they teach and in the ways they evaluate students' learning. Hierarchical beliefs concerning what constitutes knowledge, or ability, result in differentiating practices within schools (Noddings, 1990; Rosenholtz & Rosenholtz, 1981; Rosenholtz & Wilson, 1980; Simpson, 1981). The constructivist view holds that students construct different kinds of learning under varying social conditions and is contrary to the positivist belief of a fixed body of knowledge.

This study interpreted how the teacher and students played out their roles in the heterogeneous and homogeneous mathematics classrooms, the instructional methods used, and how the teacher and his students perceived and organized their interactions. This constructivist view of schooling, that all knowledge is constructed through social experiences, is based on Berger and Luckman's (1967) theory of the social construction of reality.

> Institutions are embodied in individual experience by means of roles. . . . The realization of the drama depends upon the reiterated performance of its prescribed roles by living actors. The actors embody the roles and actualize the drama by representing it on the given stage. Neither drama nor institution exist empirically apart from this recurrent realization. To say, then, that roles represent institutions is to say that roles make it possible for institutions to exist, ever again, as a real presence in the experience of living individuals. (Berger & Luckmann, 1967, p.74, 75)

Constructivist educators assume that all knowledge is actively constructed; there is no pre-existing body of knowledge that must be imparted to students with the expectation that understanding will result. Constructivism assumes that human beings are knowing subjects; that their behavior is mainly purposive; and that they have a great capacity for organizing knowledge (Magoon, 1977). The following constructivist views hold important implications for teachers of mathematics.

> 1. All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
>
> 2. There exist cognitive structures that are activated in the processes of construction. These structures account for the construction; that is, they explain the result of cognitive activity in roughly the way a computer program accounts for the output of a computer.
>
> 3. Cognitive structures are under continual development. Purposive activity induces transformation of existing structures. The environment presses the organism to adapt.
>
> 4. Acknowledgement of constructivism as a cognitive position leads to the adoption of methodological constructivism.
>   a. Methodological constructivism in research develops methods of study consonant with the assumption of cognitive constructivism.
>   b. Pedagogical constructivism suggests methods of teaching consonant with cognitive constructivism. (Noddings, 1990, p. 10)

Based on Berger and Luckmann's theory of the social construction of reality, constructivist theorists assume that knowledge is socially constructed by members of a social group. Therefore organizational practices, such as ability grouping, which define the characteristics of the knowledge produced, may be considered by constructivists as "educational situations" where "particular types of constraints were operating" (Magoon, 1977, p. 665).

Constructivists regard the formation of academic ability, in part, as a social construction. The production of homogeneous groups through social influences may adversely restrict opportunities; ability groups provide different educational experiences and limit the choices available to students and teachers for social comparison of abilities. Constructivist theory stresses the need for alternatives to traditional methods of instruction. These needs have been well-documented (Goldin, 1990; Goodlad, 1984; Noddings, 1990; Oakes, 1985, 1990; Romberg & Carpenter, 1986; Rosenholtz & Simpson, 1984).

Magoon (1977) noted that many ethnographers seek to discover what particular types of constraint are operating in the given educational environments. He wrote of the need to "observe teachers and students as active constructors of their knowledge and rules" and reported that "an account needs to be taken of external structural constraints that have a heavy influence on individual constructions" (p. 665).

Constructivist methods of research were particularly appropriate for this study because it represented an attempt to determine how students and their teacher work together to construct mathematical knowledge in classrooms variously grouped for instruction. The intensive study of classrooms through the use of qualitative methods of observation, video-taping and in-depth interviewing adhered to the tenets of Berger and Luckmann's (1967) theory of the social construction of reality. Constructivist research suggests qualitative, ethnographic-type, methods (Magoon, 1977; Noddings, 1990).

Methods

Constructivist views about mathematics and the nature of mathematical thought have direct implications for research practice (Davis et al., 1990). The constructivist perspective (i.e., that students will react in different ways toward mathematics based on the meaning mathematics has for them) influenced the research questions that guided the current study:

1. How does the delivery of curriculum and instruction differ in the heterogeneous and homogeneous mathematics classes observed?
2. How do social and mathematical interactions differ in the classes observed?
3. What constitutes meaningful mathematical learning in the various classrooms?

Because of the long experience of the researcher as a middle school mathematics teacher, the main focus of the study consisted of an interpretive, qualitative approach that provideed more intense, detailed description than could be gained solely from analyses of surveys and pre- and post-tests. However, benefits can be obtained by a combined use of qualitative and quantitative methods (Reichardt & Cook, 1979). In order to evaluate the research data in the most efficient manner and obtain a multiple perspective, data from achievement tests and surveys were embedded in the discussion of the findings.

As a mathematics teacher, this researcher's membership of that particular educational discipline influenced the theoretical perspective and research process of this study. Researchers are socialized into disciplines that have their own vocabulary, concepts, and theories (Merriam, 1988). A mathematics educator will therefore view the world through this discipline's lens. Mathematics teachers ask questions such as, "What mathematics should students learn?" and "What kind of mathematical experiences do schools need to provide to children?"

Constructivist teachers answer these questions building on the theory of symbolic-interactionists who hypothesized that individuals construct and share meanings through interpretation and interaction. These teachers view mathematics as invented or constructed by individuals, rather than as a pre-existing body of knowledge; that an interpretation of mathematical meaning is constructed by the learner, rather than imparted by the teacher. They believe that meaningful mathematical learning will not occur using the traditional, passive, pedagogical methods of teacher-directed rote learning. Goldin (1990) believed there is a need for descriptive case studies to replace controlled experimentation in the assessment of mathematics learning and teaching effectiveness because the cognitions of students are simply not comparable.

Woods (1988) suggested that ethnography can lessen the distance between theory and practice in education by helping "educators glean new insights, expand their views of reality, and balance their perspectives" (p. 98). He stated that educational ethnography could contribute to teacher professionalism because it is concerned with substantive issues important to teachers, dealing with their problems and attaching importance to their perspectives. Ethnographical techniques take the implications of teacher actions in different situations into account and "even employ(s) the concepts and language of school culture in drawing descriptions and spelling out theories" (Woods, 1988, pp. 97, 98).

Merriam (1988) asserted that qualitative study builds theory. The advantages of ethnographic-type methods of research have also been described in other ways: as allowing the discovery of truly applicable theory; allowing the researcher to understand events as they are conceived by the participants; and offering the opportunity for investigations of controversial topics, in depth, over a longer period of time (Biddle & Anderson, 1986).

However, Eisner (1991) warned that ethnography "even taken as a whole, constitutes a limited repertoire of meaning structures. Thus, to regard qualitative research as solely ethnographic is to limit what qualitative work can be" (p. 229). If social concepts and explanations are, for the most part, socially constructed, then qualitative, ethnographic-type methods were most appropriate for this interpretive study. These methods included, but were not restricted to, intensive classroom observation and videotaping, and in-depth interviewing.

This study described, in terms of constructivist theory, how the social organization of schools may affect and constrain instructional practices, thus influencing teacher and students' behaviors and perceptions. It was conducted at one middle school during an entire school year. Informants included one sixth-grade mathematics teacher and his 160 sixth-grade students who were organized in one of three ways: (a) they were placed in advanced classes, where all students achieved above the 92nd percentile as measured by the mathematics section of the fifth grade California Achievement Test; or (b) they were placed in a heterogeneous pilot class, whose students' achievement scores were representative of the entire sixth grade population; or (c) they were randomly assigned to basic heterogeneous classes that were minus the advanced students.

The study provided insight into how classroom participants make sense of their experiences as members of heterogeneous or homogeneous mathematics classes. The constructivist perspective taken by the researcher demonstrated the applicability of this theory to research on how social organization within schools affects students', and teachers', behaviors and perceptions.

Significance of the Study

The need for studying the ways that grouping methods influence students' academic potential has been well-documented (Gamoran & Berends, 1987; Slavin, 1987). "There is much more we must understand about how various ability grouping plans have their effects. A theory able to encompass the research findings is needed." (Slavin, 1987, p. 321)

Constructivist theory may help practitioners and researchers understand how children learn mathematics most effectively; it may assist educators in distinguishing meaningful learning from rote learning and also may determine what constitutes effective pedagogical practice. Too frequently, in a majority of ability-grouped classrooms, the time spent on mathematics each day is largely devoted to the mastery of outdated computational skills (Goodlad, 1984; Oakes, 1985, 1990). Attempts to use teaching strategies that involve students as active learners have been considered appropriate only for selected groups. Today many researchers consider these strategies essential for all students; effective learning depends on the active engagement of students in constructing their own knowledge and understanding (Sheingold, 1991).

Although the National Council of Teachers of Mathematics named the 1980's the "decade of problem solving," Sedlak, Wheeler, Pullin, and Cusick (1986) noted that reports of the 1980s revealed the continuing erosion of higher-order cognitive and reasoning skills among adolescents. Only six percent of students aged seventeen were able to solve multi-step problems involving simple algebra (Dossey et al., 1988). These reports suggested that mathematics be made more natural, more relevant to the lives of students, and that testing should have less threatening implications. These suggestions support the constructivist view on the teaching and learning of mathematics.

Apart from the embarrassment of the international comparative reports, they generate concern that high school graduates lack sufficient skills to be productive citizens in a democratic society (Carnegie Council, 1989). Sedlak et al. (1986) suggested that existing organizational and instructional practices fail to create academic environments that motivate the majority of young adolescents, especially middle-class youth. Oakes (1990) pointed particularly to the areas of mathematics and science in which minority and disadvantaged students scored disproportionately low. Constructivism suggests that mathematics should be perceived as part of the real world and therefore fit better into the lives of students, and that testing should have less threatening implications (Davis et al. 1990).

One of the principal reasons for mathematics under-achievement in the United States today has been attributed to the use of outmoded organizational and instructional practices (Useem, 1990; Usiskin, 1987). Often beginning in the early elementary grades, ability grouping ensures that by the middle-school years, students have been exposed to sharply differentiated curricula. The National Assessment of Educational Progress (1988) revealed that by eighth grade, students are generally assigned to one of four types of mathematics classes: remedial, basic, enriched or algebra. The practice was found to be more pervasive in the United States than in any of the other countries studied in the report (Dossey et al., 1988). This practice of grouping students by ability authoritatively conveys to students the school's belief about their potential for success (Carnegie Council, 1989; Rosenbaum, 1976).

Ability grouping has long been considered an efficient method of accommodating students' individual differences within the public school system. It first assumed educational significance when the great influx of non-English-speaking immigrants at the turn of the century, together with America's evolution from an agrarian to an industrial society, caused dramatic demographic changes. Education was considered the solution to the myriad of social problems; ability grouping was introduced to help solve the specific educational problems of that time.

As the turn of a new century approaches, the demand increases for a labor force with high technical skills. Projected demographic changes predict that the white, middle-class pool from which scientific workers have traditionally been drawn will continue to shrink, as the numbers of Hispanic, black, and poor children continue to rise rapidly in American public schools. By the year 2020 nearly one-half of all public students will be non-white (Carnegie Council, 1989).

Constructivist theory may prove useful in suggesting effective mathematical teaching practices that accommodate the needs of students with different backgrounds and with varying levels of ability. Oakes (1985) found that the conditions of classrooms resulted in poor and minority students receiving an inferior education in terms of the amount of culturally valued knowledge transmitted. Young (1971) suggested that perhaps the organization of knowledge implicit in current organizational and instructional practices is so taken for granted that educators are unable to think of alternatives. The constructivist perspective adopted in this study may help to expand our understanding of the way students learn, and may suggest new forms of pedagogical practice.

Constructivist theory recognizes that each student's knowledge is individually constructed out of the reality experienced by that student; this is unique to the individual and contextually dependent. For example, the way many educators interpret academic ability results in the construction of a particular reality experienced by students. Knowledge constructed by disadvantaged students may come from a world of negative experiences; minority and poor students are over-represented in low-ability groups. Only 17% of students experience the reality of advanced classes (Useem, 1990).

This constructivist perspective is particularly relevant to middle-school educators. Middle schools that enhance student self-esteem may be characterized by a humanistic and democratic climate, heterogeneous grouping, cooperative learning, and "activities that involve making, creating, and doing" (Beane, 1991, p. 27). The effects of a narrow curriculum and of teaching practices that emphasize memorization are likely to be especially harmful to early adolescents who are developing long-term attitudes toward learning (Becker, 1990). Constructivist theory may assist educators in understanding the nature of knowledge and may point to pedagogical practices that are more meaningful to early adolescents.

The current study helps determine the usefulness of the constructivist perspective in explaining differences in attitudes and achievement that result from variations in organizational and instructional practices. Slavin (1987) called for a theory that will encompass the findings from studies about the effects of various grouping arrangements. He noted the absence of descriptive studies that contrasted teachers' and students' perceptions in classes variously grouped for instruction. Other researchers also demonstrated the need for qualitative studies that look at the composition of heterogeneous and homogeneous classes and assess how these differences shape learning opportunities (Dawson,1987; Gamoran, 1987; Oakes, 1990). Previous researchers have tended to ignore the character of the actual teaching process; more investigation needs to be done into the teaching of heterogeneous classes "especially with regard to the way in which teachers perceive pupils differently, interact with them differently, and convey to them different conceptions of personal worth" (Ball, 1981, p.279).

Definition of terms

The following terms are essential to an understanding of this study:

Ability grouping refers to the practice of assigning students to homogeneous classes based on educators' perceptions of their academic abilities.

Cooperative groups are groups that usually contain a small number of students of mixed abilities who work together on a common assignment.

Mathematical interactions refer to the discussion that takes place between students as they attempt to solve mathematical problems.

Multidimensional classrooms are those classrooms where tasks are decentralized; students are engaged in multi-tasks, in multi-group settings.

Social constructivism refers to the belief that members of a social group, individually and collaboratively, construct and maintain a reality that influences the acquisition of knowledge; knowledge is then a product of each individual's cognitive acts.

Social organization refers to a school's method of organizing students for instruction.

Unidimensional classrooms are those classrooms characterized by students working individually on the same task.

Within-class ability grouping is the assignment of students to small homogeneous groups within the heterogeneous classroom.

Overview

The following four chapters include a review of the literature, the methodology, the findings, and the conclusions and implications. Chapter 2 links the findings from research on ability grouping to reviews of literature that examine organizational and instructional practices relevant to early adolescence. The literature review summarizes meta-analyses of research on the effects of ability grouping on achievement, includes a theoretical framework based on symbolic interactionist and constructivist theories, and discusses effective organizational practices for mathematics instruction. Chapter 3 discusses reasons for the selected research methods. Chapter 4 reports on the results of findings from the study. Conclusions and implications for further research are discussed in Chapter 5.

CHAPTER 2
REVIEW OF THE LITERATURE

Ability grouping has long been one of the most controversial issues in education; in hundreds of studies researchers have examined the effects of various grouping arrangements on students. The purpose of this review of the literature is to summarize meta-analyses of research on ability grouping, and examine recent research on organizational and instructional practices related to the special needs of middle school students, in order to provide a background of research and theory to support the present study. This review of the literature supports the view that the complexity of early adolescent development necessitates the use of specific organizational and instructional strategies.prescribed through developmental theory and the theories of social constructivism.

The following review of the literature examines the effects of ability grouping on students' achievement and social development. The literature review also examines the relationship between constructivist theory and how the social organization of schools affects the teaching and learning of mathematics, and students' mathematical self-concept.

Effects of Ability Grouping on Achievement

Ability grouping has been a controversial issue in education from the time of its inception.

> For a number of years educators have concerned themselves with altering school practices in the light of changing social needs, new scientific knowledge, and new educational theory. Among the problems encountered, the question of adapting education to the great range of individual differences at all age levels has been one of the most persistent and challenging. It is a challenge because it strikes at the roots of educational philosophy. (NSSE Handbook, 1936)

Slavin (1987), in his review of the research on ability grouping at the elementary level, stated that the advantages and disadvantages of ability grouping have been given by theorists for over 50 years and have resulted in hundreds of studies. He identified many comprehensive reviews of ability grouping that were written more than 20 years ago (Borg, 1965; National Education Association, 1968; Heathers, 1969; Findley and Bryan, 1971) and cited more recent reviews (Esposito, 1973; Persell, 1977; Good & Marshall, 1984). Slavin stated that "almost without exception, reviews from the 1920s to the present have come to the same general conclusion: that between-class ability grouping has few if any benefits for student achievement" (p. 293).

Only recently has the meta-analysis approach been advocated by researchers "in quest for a mode of analysis that would provide better-grounded inferences regarding the relationships of teaching and achievement than can be gleaned from single studies examined individually" (Shulman, 1986, p. 10). This current study has benefited from the work of those researchers who have in recent years used the technique of meta-analysis to perform exhaustive reviews of the research on ability grouping. "The meta-analysis technique developed by Gene Glass and his associates provides a more sophisticated method of combining the results of related studies" (Borg & Gall, 1989, p. 157). This method involves converting the findings of each study to an effect size. "The mean of the effect size of all studies included in the research is then calculated to estimate the typical effect of the phenomenon under study" (Borg & Gall, 1989, p.172).

Kulik and Kulik's (1982) meta-analysis of 52 studies reported the effect of ability grouping on secondary students and found that effects were largest in the 14 studies where high achieving students were homogeneously grouped for instruction. However, only 8 of the 52 studies reported statistical significance favoring homogeneous grouping. The effects were near zero in 33 comparisons of tracked and mixed ability classrooms. "The benefits of grouping tended to be slight in the area of achievement--an average of one-tenth standard deviations on achievement examinations" (p. 425).

Eleven of the 31 studies included in the meta-analysis of the effects of grouping on elementary students showed statistical significance when students were homogeneously grouped (Kulik & Kulik, 1984). Again the effects were especially favorable for academically superior students who raised achievement scores by .49 of a standard deviation. The more representative school populations raised scores by .07 of a standard deviation. "The benefits of grouping tended to be small in the typical study of achievement" (Kulik & Kulik, 1984, p. 6).

C. Kulik's (1985) meta-analysis of 40 elementary school studies and 45 secondary school studies on ability grouping, supported her earlier findings (1981, 1982, 1984) that indicated clear academic gains only for high-ability students placed in programs especially designed for them. Slavin's (1987) review excluded these kinds of studies as he regarded it likely that "characteristics of special accelerated programs for the gifted account for the effects of gifted programs, not the fact of separate grouping per se" (p. 307). Slavin stated that gifted and special education programs may be conceived of as one form of ability grouping, but they also involve many other changes in curriculum, class size, resources, and goals that make them fundamentally different from other homogeneous groups (p. 297). Slavin's view supports the constructivist approach of the present study, which will describe how the environment influences the way that students construct intellectual ability.

The time frame differs between the studies used for meta-analyses. Contrasted with the meta-analyses of others, Noland and Taylor's (1986) meta-analysis of research findings on ability grouping "is based on all research reported on ability grouping between 1967 and 1983" (p.30), in kindergarten through 12th grade classes within the United States. The major findings based on 720 measurements derived from the data of 50 studies, which were all either experimental or quasi-experimental, demonstrated that students who were ability grouped had the same cognitive outcomes as those who were not ability grouped and that there were adverse effects on the self-concept of students who were ability grouped. In contrast, C. Kulik and J. Kulik's (1984) meta-analysis, included studies which ranged from 1928 to 1970 (including some from England), and in Slavin's (1987) meta-analysis all but two of the studies in the review were published between 1936 and 1972.

Slavin (1987, 1990) described his review of the research on the effects of ability grouping on achievement in elementary schools and secondary schools as "a best-evidence synthesis" that combined meta-analysis and traditional narrative review. Borg and Gall (1989) commented that a persistent criticism of the meta-analysis approach to reviewing previous research is the inclusion of data from poor studies. In Slavin's reviews (1987, 1990) he examined only what he defined as methodologically sound studies that differentiated between achievement gains made by students in various types of grouping arrangements; he examined 14 studies of comprehensive ability grouping in elementary schools (1987).

The results of the meta-analysis "were surprisingly clear cut for most types of grouping" (Slavin, 1987, p. 321). Evidence drawn from randomized and matched equivalent studies did not support the assignment of students to self-contained, ability grouped classes. Although the research was unclear on the achievement outcomes of students homogeneously regrouped for mathematics and reading, some gains were made in the upper elementary grades when students were ability grouped for mathematics within the classroom. Slavin's (1987) review of the research has suggested that these within-class groups be used to reflect students homogeneity on specific skills, that they be flexible enough to allow for easy reassignment of students, and that the number of groups be small enough to allow teachers time to provide direct instruction to all groups.

In a meta-analysis of research studies, Henderson (1989), compared overall achievement, high-ability student achievement, and combined average- and low-ability student achievement for elementary students heterogeneously and homogeneously grouped for instruction. The findings have corroborated those of Kulik and Kulik (1981, 1982, 1984, 1985); homogeneous grouping raised achievement scores of high-ability students, while heterogeneous grouping slightly favored the average- and low-ability students. However, as found in Kulik and Kulik's meta-analyses, effect sizes were not significant.

In his (1990) meta-analysis of research findings on ability grouping in secondary schools, Slavin combined the results from 29 studies. Of all the reviews, this review provided the strongest support for the current study as it concentrated on studies, mostly involving grades 7 through 9, that compared between-class ability grouping to heterogeneous placement. Findings from Slavin's study have contradicted earlier studies (Henderson, 1989; Kulik & Kulik, 1981, 1982, 1984, 1985) that reported significant achievement gains for the brightest students when they were homogeneously grouped. Zero effects of ability grouping have been found for all ability levels in all subjects, except social studies where heterogeneous grouping was superior (Slavin, 1990).

Meta-analysis has proven to be a "sophisticated" method for combining the results of related studies (Borg & Gall, 1989, p.157). Recent researchers of the effects of ability grouping on achievement have used this technique to assess the findings from several hundred studies (Kulik, C, 1985; Kulik & Kulik 1982, 1984; Henderson, 1989; Noland & Taylor, 1986; Slavin, 1987, 1990). From studies spanning over 70 years, conclusions indicated that while some small gains were reported for students homogeneously grouped in special programs, overall no academic advantages were gained as a result of ability grouping.

Gamoran (1987) suggested that a major limitation of many reviews is that they do not indicate the manner in which ability grouping affects student achievement; the studies do not distinguish between types of classroom organization (i.e. how students are arranged between classes) and instruction (i.e. the processes that occur within classes). Slavin's research (1987) also demonstrated a need for a better understanding of the effects of various grouping arrangements on instruction. These researchers have indicated the need for studies similar to the current study.

> A theory able to encompass the research findings is needed, In the absence of descriptive studies capable of contrasting teachers' and students' behaviors and perceptions in alternative grouping arrangements, we can only speculate about how the effects of various grouping plans may be produced. Such studies are clearly needed. (Slavin, 1987, p. 321)

When meta-analysis techniques were used to determine the effects of ability grouping on students' attitudes and self-concept, results again were mixed. C. Kulik and J. Kulik in their several meta-analyses described these effects in equivocal terms; homogeneous grouping is often beneficial for high-ability students, may have a slightly positive effect on the self-concept of low-achievers, but has little effect on average students' self-concept (1982, 1984, 1985). Other researchers using meta-analysis were less ambiguous. Henderson, for example, also reported positive effects on student attitudes for high-ability students in homogeneous classes but strong negative effects for average and low-ability students in homogeneous classes. Other meta-analyses of research on ability grouping reported consistent negative effects on students' self-concept, especially for students placed in the lowest groups (Noland & Taylor, 1986; Slavin, 1987).

The current study, which contrasts the behaviors and perceptions of a mathematics teacher and his sixth-grade students in classes variously grouped for instruction, will be guided by questions generated from this review of the literature. Several of the studies examined in the continuation of this review apply constructivist theory to educational contexts, and demonstrate that the complexity of early adolescent development necessitates the use of certain organizational and instructional procedures.

Theoretical Framework

The broad theoretical framework of symbolic interactionism guided much of the research reviewed in this chapter. Symbolic interactionism was named and developed by Herbert Blumer in the late 1930's, drawing largely on the earlier work of G. H. Mead. Focusing on the premise that humans act towards things on the basis of the meaning those objects have for them, symbolic interactionism analyzes human social behavior; symbolic interactionists study the subjective aspects of human life in which the actors define and interpret each other's acts as they move through life. Symbolic interactionists are concerned with understanding how the processes involved in symbolic interactionism occur and how individuals are able to take one another's perspective and learn meanings and symbols in concrete instances of interaction (Jacob, 1988).

Berger and Luckmann's (1967) view of society as subjective reality applies this understanding to the level of subjective consciousness, the individual's social construction of reality. Their work builds a theoretical bridge from the work of Blumer and Mead to the constructivist view that all knowledge is socially constructed; that subjective knowledge results from the reality created for individuals through social interactions and human activity .

Constructivist Theory

Basic premises of constructivist theory have long been recognized in educational research. Mead (1934) recognized the crucial role of small-group activities for children; they learn, in social groups, how to construct knowledge about things and events. He described how children's personalities develop in relation to others in the group; they internalize the attitudes of others and become aware of their identities. Groups train members through assisting in their psychological development; by providing the context in which intellectual and social development takes place (Olmstead & Hare, 1978).

Piaget's work parallels that of Mead at many points (Olmstead & Hare, 1978). He has been called "the great pioneer of the constructivist theory of knowing" (von Glasersfeld, 1990, p.22). Piaget's basic work on the child's first two years of cognitive development has been consolidated in a book called The Construction of Reality in the Child (1951).

> Whether reality is constructed or discovered has vast implications for all facets of a theory of knowledge. . . . For mathematics education the former implies that the child must actively construct its own foundation and its own mathematics...The latter implies that knowledge is already out there and must somehow be inserted into the child. (Richards & von Glasersfeld, 1980)
> .

Behind Piaget's theory of cognitive development a model of change and development is at work. Constructivist educators believe that as students progress through the assimilation of, and adaption to, further mathematical experiences, their knowledge progresses; this progression is not, however, in any ultimate sense toward a definitive goal. Piaget's model insists on the possibility of alternative solutions to a single problem; it is this possibility of diversity that is the foundation of constructivism. Berger and Luckmann (1967) acknowledged the influence of Piaget's view of the massive reality of the child's world on their theory of the social construction of reality. See pages 197, 201, 203, 205, and 208 for examples of this influence.

Berger and Luckmann's (1967) theory has influenced many educational studies that attempt to determine how school activities are understood by the participants. In applying constructivist theory to education, Magoon (1977) cited several dozen published constructive ethnographies that provide rich description of educational phenomena. Many studies reviewed in this chapter have also relied heavily on Berger and Luckmann's (1967) theory in developing their propositions concerning the effects of the educational environment on students' self-perceptions (Cicourel & Kitsuse, 1963; Rosenholtz & Rosenholtz, 1981; Rosenholtz & Simpson, 1984; Simpson, 1981).

Magoon (1977) stated that the constructivist approach is most appropriate when researchers wish to "observe teachers and students as active constructors of their knowledge and rules (and when) an account needs to be taken of external structural constraints that have a heavy influence on individual constructions" (p.665). The effects of ability grouping on students' construction of mathematical knowledge is examined in this review of the literature; it is an example of such an educational constraint.

The current study will work within the dialectic epistemology of Mead (1934) and Blumer (1969), which has more recently been articulated through the constructivist views of Berger and Luckmann (1967). As participants in classroom interactions, teachers and students create a shared reality. Yet surprisingly little is known about how knowledge is conceived in the reality of classroom experience. "Particularly important are the pedagogical principles underlying the implementation of innovation and the ways in which the objective existence of new knowledge structures acts back on, and transforms, the teacher's and pupils' subjectively apprehended reality" (Esland, 1971). This review of the literature suggests the need for further research.

The main emphasis of the current study is an investigation of how the social organization of knowledge is structured within individual classes. Berger and Luckmann's (1967) views on how reality is socially constructed are, therefore, highly applicable to the current study. The ideas of Blumer (1969) and Mead (1934) are also supportive to this study "because they contain the assumptions which lie behind the methodologies and reality tests of teaching, they powerfully define, for teachers and pupils, what 'being educated' means" (Esland, 1971).

Constructivist Views on the Teaching and Learning of Mathematics.

Constructivist theory holds several views. "Those who label themselves as constructivists have differences of opinion on theoretical issues and express preferences for some strategies over others" (Davis, Maher & Noddings, 1990, p.3). The constructivist views presented in this review of the literature will demonstrate the applicability of constructivist theory to the teaching of mathematics and to how young adolescents learn mathematics.

The constructivist view of mathematical learning assumes that students are active reorganizers of their mathematical experiences (Wood, Cobb, & Yackel, 1991). This view is influenced by Piaget's developmental theory that mathematical knowledge is actively created by children through a process of cognitive conflict, reflective abstraction and conceptual reorganization (Piaget, 1970). Constructivism also assumes that members of a group cooperatively establish mathematical ideas (Blumer, 1969).

These views are, therefore, in conflict with traditional approaches that regard learning mathematics as the following of explicit rules that are related to a fixed body of knowledge. "If we regard the doing of mathematics as involving complex processes that call for the use of heuristics and analyses, then another kind of learning activity becomes appropriate" (Davis, Maher, & Noddings, 1990, p. 2). Constructivists recognize that students must be given opportunities to construct knowledge and assimilate accurate mathematical understanding. Cognitive research indicates that it is essential to distinguish between meaningful learning and rote learning.

An example of the need to give students opportunities to construct mathematical knowledge was provided by Maher and Davis (1990). Two fifth-grade boys were videotaped as they worked together to solve a problem about pizza slices. After much discussion and the use of pattern blocks and sketches, the boys reached the correct answer; when one of the boys attempted to use paper and pencil methods, the answer was incorrect. The teacher came to the boys as they finished the problem but was dissatisfied with their reasoning methods. She spent time working with them and was finally successful in directing them to the solution she wanted. One year later the boys were given the same problem and were again observed solving it, this time independently. They indicated that although they remembered the problem, they could not remember how they had solved it. It is significant that again they solved the problem by their own methods; the teacher's solution had no meaning for them and was not durable over time.

In a research project based on the constructivist perspective a second-grade teacher changed her beliefs about learning and teaching mathematics (Wood, Cobb, & Yackel, 1991). Prior to the project the teacher had taught by adhering closely to the teacher's manual for the textbook. Instead of teaching rules and rote learning, the teacher introduced organizational and instructional practices that changed mathematics into a meaningful activity. Open-ended instructional activities were created that allowed for a variety of solution methods. Classroom organization encouraged pair collaboration and active whole-class discussions; these various grouping arrangements provided opportunities for students to exchange ideas, offer explanations, justify their thinking, and negotiate mathematical meaning that was accepted by members of the class. The teacher came to realize that teaching and learning mathematics involve social interactions in which students are actively engaged in creating mathematical meaning. .

Recent reports document the need for the constructivist perspective in the teaching of mathematics. "At every level of schooling, and for all students, reform documents recommend that mathematics students should be making conjectures, abstracting mathematical properties, explaining their reasoning, validating their answers, and discussing their own thinking and the thinking of others" (Lampert, 1990, p. 32). Lampert described the experimental methods she used when teaching fifth grade mathematics classes, basing them on the ideas of Polya (1957) who asserted that mathematicians must be willing to question both observations and generalizations. This contrasts with traditional notions of mathematical practice in which mathematics is viewed as a fixed body of knowledge.

Mathematics for the Young Adolescent

Usiskin (1987) argued that at this period of cognitive development a repetitive curriculum that spends a great deal of time reviewing work of earlier grades is boring and unmotivating. He assessed that less than 25% of seventh grade textbooks contained material new to students. Eccles and Midgley (1989) have pointed to evidence that first year mathematics at junior high schools requires lower-level cognitive skills than classwork at the elementary level.

Goodlad's (1984) study of schools has also revealed that the dominant theme of middle school mathematics classes is reviewing basic operations. Almost the same skills were being taught in successive years with only slight increases in the level of difficulty. Some teachers paid attention to consumer mathematics and a few schools offered algebra in the eighth grade, but for the most part the textbooks used dominated the curriculum with basic operations. These mathematics teachers stated that they wanted their students to have higher-order thinking skills. Why then "did so few mathematics teachers in our sample appear to get beyond a relatively rote kind of teaching and textbook dependency not likely to develop powers of critical reasoning" (Goodlad, 1984, p.210)?

Findings from the National Assessment of Educational Progress (Dossey et al., 1988) indicate that students appeared to learn mathematics at the rote level without understanding the concepts related to the computation. Although students enter school with a well-developed intuitive system for problem solving, after several years in school this intuitive thinking is abandoned in favor of a memorized set of rules for problem solving (Baroody & Ginsberg, 1990).

An example of rote learning without understanding was provided in a study that described how a young adolescent dealt with a problem that involved converting 3 feet 6 inches from one unit of measurement to another (Baroody & Ginsberg, 1990). She had difficulty in remembering the steps and when the interviewer tried to offer assistance by asking if there was another way of doing it, she replied abruptly that this was the way it had to be done. Even though she knew that 6 inches equals .5 feet, her belief that there was only one way of solving the problem prevented her from using that knowledge. After examining case studies of mathematics and science teachers, Tobin and Fraser (1989) remarked "the emphasis is on learning facts and memorizing algorithms to solve problems without necessarily understanding why the algorithms work" (p.320).

Studies of mathematics and science classrooms describe bland, traditional, teacher-directed environments (Goodlad, 1984; Oakes, 1985; Trimble, 1988). The teaching and learning that takes place has not changed in the last 20 years. Welch (1978) reported then that

> in all math classes I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked by the teacher or a student at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to working on the homework while the teacher moved about the room answering questions. The most noticeable thing about math classes was the repetition of this routine. (p.6)

Research has emphasized the importance of interactions within the classroom for promoting mathematical knowledge. This was demonstrated in Evertson, Anderson, Anderson, and Brophy's (1980) study in which 29 mathematics teachers were observed in 11 junior schools. Subjective rating scales and objective counts of discrete behaviors were used by observers to determine which teachers were most effective and why.

Effective teachers were more active and used less time in seatwork; about 80 % of classtime was used for lecture-demonstration, class discussion and seatwork. They spent more time asking many different types of questions. Types of responses depended on the level of class ability; students in the high-ability classes responded more positively and had a greater level of demand. The number of student-initiated questions and comments correlated positively with student achievement in mathematics. In the low-ability classes even the effective teachers had to devote more time to dealing with behavior problems. Studies showed that successful mathematics teachers were active, well-organized, and strongly academic, emphasizing active whole-class instruction and devoting less time to seatwork. Effective teachers asked many questions, both high-and low-order, and managed their classes efficiently (Evertson et al. 1980; Good, Grouws, Mason, Slavings, & Cramer, 1990).

Leinhardt (1989) observed for a period of more than three months, four mathematics teachers whose student growth scores had been consistently high for three years. The study noted that expert mathematics teachers constructed lessons that built upon and advanced material introduced in previous lessons, that consisted of well-defined goals, and that moved fluidly from one activity to another. These teachers used multiple representations and divided topics into manageable pieces in order to minimize student confusion. The teacher-based presentation "is perhaps the most important teaching segment and the one that must be most carefully constructed because that is where the explanation of new mathematical material usually takes place" (Leinhardt, 1989, p. 54).

Studies are often divided on the effectiveness of whole-class or small-group instruction (Evertson et al., 1980; Gerleman, 1987; Janicki & Peterson, 1981). Gerleman found small-group instruction less effective than whole-class instruction because less time was spent on developing concepts, and students rarely used higher-order cognitive processes. The study indicated that the apparent purpose of small-group work was to accommodate differences in students' achievement.

Good et al., (1990) studied the extent to which the structure of small-groups and instructional practices affected student learning in fourth through sixth grade mathematics classes variously grouped for instruction. Observers rated the small-groups based on ability least effective; little quality work was produced. Students seldom worked together even though they were of comparable ability. Observers reported that teachers who used small heterogeneous work-groups for part of the period often assign more higher-level tasks and that this stimulated better student interaction and cooperation.

Although whole-class lessons include a greater use of manipulative materials such as connecting cubes and pattern blocks, and are generally rated as more meaningful with more emphasis on higher-level cognitive skills than the small-group lessons, students are more likely to be off task. Effective teaching practices combine the use of whole-class instruction and heterogeneous work groups. The study noted that small-group cooperative learning activities rarely occurr in ability grouped classrooms (Good et al., 1990). This may help to explain Janicki and Peterson's (1981) findings that high-ability students preferred individual seatwork to small-group work even though small-group work was particularly appropriate for developing higher cognitive goals (Good et al. 1990; Graybeal & Stodolsky, 1985).

Good et al., (1990) called for observational research that describes the different forms of small-group teaching, explains how teachers provide instruction within each form, and depicts why some grouping arrangements are more effective than others. Also an exploration is needed "of how various small-group formats affect important factors in mathematics teaching (e.g., the need for active teaching and active learning, an emphasis on conceptual understanding, variety in lesson presentation) that likely contribute to student outcomes" (p. 758). Good et al., (1990) suggested that in future studies it will be important to explore teacher beliefs about expected outcomes, student diversity, and the value of cooperative activities, as well as to examine instructional processes.

Other researchers have suggested the importance of cooperative learning in accommodating varying levels of student ability in classrooms (Goodlad, 1984; Johnson &

Johnson 1975; Oakes, 1985; Slavin, 1991). In his synthesis of research on cooperative learning, Slavin (1991) listed consistent positive effects on students' self-esteem, attitude to school, intergroup relationships, and the ability to work cooperatively. Middle school students are at the stage of social development when cooperative learning has been shown to be an especially appropriate instructional strategy. Interaction with peers plays an important role in the development of students' self-concept through encouraging equality and respecting diversity. However, while much attention is paid to the effects of teacher-student interaction there is a tendency to ignore the instructional potential of student-student interaction (Johnson & Johnson, 1975).

The amount of academic interaction between students is determined. by the way teachers structure learning goals. When students work on the same tasks individually in competition for the highest grade, one student's success depends upon another's failure. "This negative interdependence influences the way students interact. They often treat each one another as rivals and are unwilling to share information or resources" (Oakes, 1985, p. 209). Studies have suggested that teachers tend to avoid heterogeneous small-group work, favoring ability grouping and the associated traditional, recitation-type, instructional practices; methods that have resulted in students being bored and passive (Good et al, 1990; Goodlad, 1984; Oakes, 1985; Trimble, 1988; Wilson & Schmits, 1978).

Artzt and Armour-Thomas (1990) examined the roles of cognition and meta-cognition in the problem-solving behaviors of seventh grade students cooperatively grouped for mathematics. Coders studied videotapes of students as they worked together and noted the benefits as students became actively involved in mathematical learning through discussing ideas and revising opinions.

Other recent studies revealed the importance of active teaching and active learning in promoting cognitive development; this can result from small-group work or from whole-class instruction that actively engages all students through the use of higher-level questioning techniques (Gerleman, 1987; Good et al., 1990; Janicki & Peterson, 1981).

Recent reports have revealed that students in the United States are doing poorly in mathematics and science when compared to students from other industrialized nations (Dossey et al., 1988; McKnight et al., 1987; National Council of Teachers of Mathematics, 1989; National Research Council, 1989). After a decade of advocating problem solving as an important mathematical tool, only 6% of all 17 year-olds can solve a multi-step problem involving simple algebra (Dossey et al., 1988). Constructivists believe that

> They (students) find ideas in working purposively with concrete objects, in talking with each other, in sharing with their teachers. Mathematical growth, like ethical development, is varied and complex; it is under continuous construction and depends, ultimately, on whether students care about mathematics. (Davis, Maher, & Noddings, 1990, p. 191)

Given the inadequate record of student performance in recent years it is imperative that mathematical environments be created which stimulate students' desire to learn.

Mathematical Self-concept during Early Adolescence

Studies have shown steep declines in academic self-esteem during early adolescence; this decline in self-esteem is particularly apparent in students' perceptions of their mathematics ability as they progress from sixth to seventh grade (Eccles, Adler, Futterman, Goff, Kaczala, Meece, & Midgley 1983; Eccles & Midgley, 1989; Stipek & MacIver, 1989;). When a mathematics attitude survey was.administered to 714 ability-grouped seventh graders in order to determine the relationship between students' attitudes to mathematics and mathematics achievement, findings revealed a strong correlation between mathematics self-concept and mathematics anxiety (Brassell, Petry, & Brooks, 1980).

Top-ranked students in high-ability classes had the greatest self-concept; low-ranked students in average classes had the lowest self-concept scores. The average self-concept scores and anxiety scores for each class showed an overlapping trend across ability groups. The study suggested that anxiety and self-concept are not only inversely correlated to each other but may also be related to the learning context; being the low-achiever in a high ability class may undermine self-concept, raise anxiety and reduce achievement. The findings indicated that teachers should provide curriculum experiences that reduce anxiety and increase mathematics enjoyment at this critical period of attitude formation in students.

Anxiety levels were measured in a study of 1,479 fifth- and eighth graders attending three types of schools at different academic levels (Schwarzer & Schwarzer, 1991). This study also found that students of low-ability in good classes feel worse that students with high-ability in poor classes. "The most important environmental source of well-being in school is the social context . . . the social network can provide support; on the other hand it delivers information about the achievement distribution that can have a debilitating effect on the students" (Schwarzer & Schwarzer, 1991, p. 308).

Value perceptions play a critical role in determining students' intentions to enroll in advanced mathematics classes. Self-concepts of mathematical ability and the subjective value of mathematics are predictive of performance in mathematics and students' enrollment plans (Meece, Wigfield, & Eccles, 1990). In a longitudinal study of 668 mathematics students in grades 5-12, Eccles et al.,

(1983) also noted that students' interpretations of reality--that is self-concept of ability, attributions, and perceptions of the beliefs of teachers and parents--were the most important determinants of expectancies, values, and course plans. They proposed an integrative theoretical model of achievement behaviors that built on the work of decision-making, achievement, and attribution theorists.

Furthering the work of Eccles et al. (1983) and defining achievement behavior as the intention to study more mathematics, Ethington's study (1991) of eighth grade mathematics students demonstrated that expectations for success and task value had stronger influences on boys achievement than that of girls; parental expectations in mathematics were greater for males than females. The way students interpret the socio-cultural situation of mathematics classrooms may influence students' perceptions of task value and expectations for success (Eccles et al., 1983; Ethington, 1991).

During the early elementary years students' academic self-concept is at its peak, declining in subsequent years, with a particularly steep decline in early adolescence (Bloom, 1976; Eccles & Midgley, 1989; Marsh, 1989; Rosenberg, 1965; Stipek & MacIver, 1989). The self-esteem of early adolescents is particularly vulnerable to the many physical, social, emotional, and intellectual changes experienced during this stage of development. Effective organizational and instructional techniques are essential in promoting the personal, social, and academic self-concept of early adolescents.

The view that self-concept is multi-faceted, hierarchically organized, and becomes increasingly differentiated with age has important implications for middle school educators (Marsh, 1989; Shavelson, Hubner, & Staton, 1976). Shavelson defined general self-concept as academic or nonacademic. Academic self-concept was divided into self-concepts in specific content areas, for example mathematics and language arts. Nonacademic self-concept was divided into physical self-concept and subdivided again into self-concepts of physical ability and physical appearance, and emotional and social self-concept. Shavelson suggested that as students increase in age and experience, self-concept becomes increasingly differentiated.

Marsh's study (1989) examined multiple dimensions of self-concept during early adolescence; the findings generally supported Shavelson's propositions. The study demonstrated that students form self-concept in specific areas by comparing their own abilities in those different areas and by a comparison with the abilities of their peers. When students incorporated feedback from others on their various skills and abilities into the formation of their self-concept it became more differentiated. The study revealed that self-concept declined during early adolescence; boys had higher self-concept than girls at this time, particularly for self-concept of appearance.

Although Marsh found that teachers were reasonably accurate in predicting how students would respond to questions on academic self-concept, when the teachers were asked to estimate both academic achievement and academic self-concept, the two constructs were indistinguishable; teachers believed that students have relatively uniform self-concept across multiple dimensions. This did not agree with Marsh's findings that demonstrated the large extent to which students differentiated between the multiple dimensions of self-concept. Marsh indicated that future research is needed to identify what characteristics of organization and instruction affect self-concept, to develop theoretical perspectives consistent with those effects, and to explore the implications of these theoretical and empirical findings.

Other studies of students' self-perception have reported an age-related decline in students' judgments of their academic ability (Bloom, 1976; Dawson, 1987; Kifer, 1973; Rosenholtz & Rosenholtz, 1981; Simpson, 1981; Stipek & MacIver, 1989). In an early study of two groups of students in a middle-class suburb of Chicago who achieved either in the top or bottom fifth of their classes, Kifer (1973) followed the relationship between academic self-concept and teachers' grades with students from grades 1-8, using

Brookover's (1964) measurement of self-concept. Very slight differences in academic self-concept were found between the high- and low-achievers in grade two, with increasing differences at each subsequent grade level; there were major differences in self-concept for high-and low achievers at eighth grade. Kifer also studied the relation between academic self-concept and teachers' grades for the entire group of students in grades five and seven; the correlation was +.23 at grade five and +.50 at grade seven. Differences in students' academic self-concept appeared adversely affected by the cumulative feedback received from grades, teachers, and peers.

Bloom (1976) agreed that academic self-concept is likely to be based on students' perception of their previous school history. He found that students' self-concept correlated·with teachers' grades rather than standardized achievement test scores, noting that the frequency of teachers' assessments constantly emphasized students' relative standing in the class. Bloom believed that adolescents' self-concept of ability is in large part dependent on students' perception of their relative achievement, as perceived through continual teachers' assessments, during the critical elementary and middle school years.

Other researchers have taken the view that while students' perceptions of ability change over school years, perception may be more related to specific situations than was previously thought (Rosenholtz & Rosenholtz, 1981; Simpson, 1981). They pointed out that if students internalized years of feedback, older students would become less affected by organizational practices. However, studies show that as early adolescents learn to interpret situational clues about their ability they become more susceptible to classroom differences and influences.

Effects of Ability Grouping on Students' Mathematical and Social Development

The organization of instructional groups has been shown to be influential in fostering the development of both academic and non-academic self-esteem. Ability grouping in middle-level schools may also affect the cognitive and social development of students.

Ability Grouping and Mathematics

Research has indicated that traditional methods of organizing mathematics classes at the elementary and middle school levels are responsible for mathematics under-achievement in the United States today (Eccles & Midgley, 1989; Stipek & MacIver, 1989; Useem, 1990; Usiskin, 1987). By sixth, or seventh grade, students have been exposed to sharply differentiated curricula in differing social environments. Findings from tudies have indicated that students internalize the school's definition of their mathematical ability through their perceptions of the classroom environment. Students' perceptions of mathematics, that is the difficulty and value of the task and their level of success in doing it, determine their self-perceptions of mathematical ability.

Studies have suggested that schools effective in promoting students' mathematical achievement are those whose educational climate and curricular organization demonstrate high expectations for students by encouraging them to take higher-level mathematics courses (Spade, Vanfossen, & Jones, 1985). Yet less than 17% of mathematics students are in the advanced track essential for placement in college level science courses. By eighth-grade students in the United States have typically been assigned to one of four types of mathematics classes; remedial, typical, enriched, or algebra (Useem, 1990).

In many other countries, courses similar to eighth grade algebra are provided at grade seven for all students. "The median Japanese seventh grader scores at a level equal to the 95th percentile of U.S. eighth graders" (Usiskin, 1987, p. 431). Usiskin reported that current practices regarding the placement of students in algebra are not logical, age rather than ability being the criteria for placement; sixth graders cannot take algebra because only exceptional seventh graders are permitted to do so under extenuating circumstances; although eighth graders scoring at the 60th percentile may be permitted to take algebra, this would not be considered a high enough score for a younger student.

Reuman (1989) noted differences in achievement expectancies for sixth grade students who were ability grouped for mathematics. His study suggested that ability grouping constrains the choices available to students and teachers for social comparison of abilities, thereby influencing the frame of reference that students use for self-assessment and that teachers use for assigning grades. Expectations for success may be more likely in heterogeneous classrooms. Homogeneous classrooms provide automatic excuses for failure because they tend to be overly competitive. In heterogeneous classrooms students have multiple levels of ability against which to compare themselves. Bloom (1976) noted that self-concept of ability is the most significant predictor of student achievement.

Studies also warned of other negative consequences of ability grouping for mathematics. Hallinan and Sorensen (1987) indicated that the sex of students influences assignment to ability groups for mathematics instruction. In a study of students in fourth through sixth grade they found that girls were more likely to be mis-assigned to lower groups, groups which had lower academic expectations for students. Oakes (1985, 1990) pointed to the disproportionate numbers of disadvantaged and non-Asian minority students in low-ability mathematics classes.

Vanfossen, Jones, and Spade (1987) noted that even high-ability students are less likely to be assigned to advanced classes if they are of low socioeconomic status. Useem (1990) indicated that students in advanced mathematics classes have greater educational and career opportunities. Many studies have reported the relationship between race, socioeconomic status, and placement in mathematics classes.

Social Development, Friendships, and Ability Grouping

Many studies have indicated that negative behaviors are often associated with early adolescence (Carnegie Council, 1989; Eichhorn, 1984). During early adolescence youngsters enter a period of trial and error, trying out new roles in an attempt to secure self identity. They face unprecedented pressures in a society of conflicting values. Increasingly early physical maturation has been associated with these problems of early adolescence. Biological precocity may change the young adolescents body image; they may become sexually active, seeing themselves as adults with adult privileges and responsibilities (Eichhorn, 1984). "Young adolescents are at far greater risk for self-destructive behaviors - educational failure, drug and alcohol abuse, school age pregnancy, contraction of sexually transmitted diseases, violence - than their age group ever was before" (Carnegie Council on Adolescent Development, 1989, p. 13).

The prevalent practice of grouping students by ability appears to be least appropriate for early adolescents experiencing the physical, cognitive and socio-emotional changes associated with maturation. In these settings subtle influences encourage students' cognitive, physical, and social homogeneity at a time when heterogeneity is at its peak. Because of the tremendous differences in physical maturation during early adolescence, reality is very different for students at different stages of development; some are perceived as adults and are expected to exhibit correspondingly adult-like behaviors; others are still perceived and treated as children (Blyth, Simmons, & Zakin, 1985; Brooks-Gunn, Petersen, & Eichhorn, 1985).

Ball (1981) investigated the effects on early adolescents when organizational practices were changed from homogeneous to heterogeneous grouping. Teachers found that the experience of being in mixed ability groups tended to slow down social maturation; students' period of childhood appeared to be extended. The onset of adolescence was retarded in the heterogeneous as compared with the homogeneous groups. Educators should consider the possibility that negative behaviors could be reduced as a result of schools' organizational practices (Ball, 1981).

Studies have indicated that school organization has a significant effect on the social interactions of early adolescents (Ball, 1981; Damico, 1985; Hallinan & Tuma, 1978; Metz, 1983; Schofield, 1989). "During early adolescence, as the individual becomes disaffected from the family, the peer group emerges as an emotional anchorage, an experiential educator of socialized attitudes and behaviors" (Tedesco & Gaier, 1988, p. 125). Peers' influence on classmates' sense of self has been found to be especially salient during the middle school years (Berndt, 1990); this influence may be positive or negative, depending on the intrinsic nature of the peer group. The quality of peer interactions is frequently determined by educators through the use of varying types of grouping arrangements.

Friendships choices made during the middle school years may be influenced by the physical, social and cognitive changes experienced in early adolescence . Tedesco and Gaier (1988) have suggested that at this period of development, selection of friends is more often determined by physical and achievement qualities than at other times. Berndt's (1990) findings from two studies on relations between adolescents' peer relationships and their self-esteem and social status, while stressing the importance of peer influences, showed that academic achievement and classroom behavior were only weakly related to students' perceived social status and friendships. These views may suggest that homogeneous grouping is least appropriate for the social development of middle school students.

Early adolescents may feel more comfortable turning to their peers, rather than adults, for understanding. They become less dependent on parents, sharing intimate thoughts and feelings with friends. Intimacy is important in promoting discussion and support for their physical and emotional experiences (Berndt, 1982; Blyth & Traeger, 1983). "Intimate conversation can contribute to adolescent self-esteem . . . by showing adolescents that another person respects their ideas and wants their advice" (Berndt, 1982, p. 1450).

In a study that attempted to show that intimate friendships contributed to students' self-esteem two groups of sixth grade students were compared (Mannarino, 1978, 1979). The groups were matched with respect to intelligence and popularity. The group of sixth graders that named the same person as best friend for several weeks and had many intimate conversations and interactions with that one person, rather than a group of friends, reported higher self-esteem than the comparison group. In commenting on this study, Berndt (1982) stated that it was unclear whether intimate friendships caused the self-esteem of these sixth graders to rise, or whether those students who already possessed high self-esteem were preferred as best friends. Berndt (1982) expressed concern that little attention has been given to developmental changes in friendships and the processes responsible for those changes. "If friendships are particularly significant during early adolescence, either the selection of similar friends or the production of similarity through social influence might be particularly obvious during this period" (p. 1455)

Effects of school organization have implications for students' social development. Grouping policies, "or the production of similarity" (Berndt, 1982), may either develop or restrict opportunities for students to interact with others from diverse backgrounds. For example, although educators and parents generally support interracial schooling, teachers beliefs concerning the need for ability grouping often cause schools to become re-segregated, thus limiting social interactions between students of different races (Metz, 1983; Oakes, 1985; Trimble, 1988). Where organizational practices increase contact between these students black and white students become more friendly (Schofield, 1989).

Damico (1985) compared organizational effects on friendships patterns at two middle schools. One was organized on the pattern of a traditional junior high school; students were ability grouped for instruction and teachers used traditional teaching practices, lecture, discussion and seatwork. In the second school the teachers used a variety of teaching methods and students were variously grouped for instruction, sometimes working individually and other times in cooperative groups. The study has suggested that classroom experiences are an important vehicle through which students learn about themselves and others.

Students at both schools used a check list to record their social interaction and provide a picture of social networks. Greater social isolation between black and white students was reported at the junior high-type school; at the second school two-thirds of black males were included in the social networks of white males and females. The practice of ability grouping restricts both the number and variety of student contacts; this is deleterious to the promotion of positive interracial interactions. (Damico, 1981, 1985; Dubois & Hirsch, 1990; Metz, 1983; Oakes 1985).

Hallinan and Tuma's (1978) longitudinal study related the social influence of classroom practices to relationships between students in fourth, fifth, and sixth grades. They found that intimate friendships were less affected by classroom organization than weak friendships. Emphasizing the major role teachers play in the social development of students, they stated that "the way teachers group students for instructional purposes, as well as their pedagogical techniques have a powerful influence on children's affective friendships" (p. 280). Several other studies have also pointed to the necessity of physical proximity in promoting friendships (Berndt, 1982; Damico, Bell-Nathaniel, & Green, 1981). Friendships result when students "have an opportunity to interact with each other and discover those things they share in common . . . not only do individuals need to be in the same environment, but they must actually interact with each other over time" (Damico et al., 1981).

Studies have indicated that the effects of organizational and instructional practices on students' social development have important implications for improving relationships between students from diverse backgrounds (Ball, 1981; Goodlad, 1984; Oakes, 1985; Rosenbaum, 1976; Schofield, 1989). When students are ability grouped the tendency is for them to make most of their friendship choices within their group (Ball, 1981; Goodlad, 1984; Oakes, 1985; Rosenbaum, 1976). Hallinan and Sorensen's (1985) study supported the view that membership in the same ability group increases this likelihood. Their density analysis showed that the ratio of the number of best friend choices made within an ability group to the number of best friend choices possible within the group increased, implying that these groups become more cohesive over time as students increasingly narrowed their circle of friends to their ability group.

> The assignment of students to ability groups can foster a stratified friendship network. If ability grouping is correlated with race or social class, then ability grouping promotes friendships among students with similar background characteristics and may encourage race, ethnic, and social cleavages in the classroom. (Hallinan & Sorensen, 1985, p. 499)

Middle school students who moved from heterogeneous to homogeneous environments experienced a decline in the number of cross-race friendships (Schofield, 1989). Conversely, Ball (1981) noted the effects on students who changed from homogeneous to heterogeneous groups. He indicated that this did not reveal dramatic increases in friendships across social class and achievement abilities but did show less antagonism between students, and none of the intractable, negative, stereotyping that had existed between the homogeneous ability groups (Ball, 1981). Attitudes of students toward those from other backgrounds are more positive when their educational environment encourages social interactions (Ball, 1981; Oakes, 1985; Schofield, 1979).

Many studies have shown that ability grouping tends to result in poor and minority students being grouped together, causing those students already disadvantaged to feel inferior (Goodlad, 1984; Oakes, 1985; Rosenbaum, 1976). This is especially damaging during the middle school years, when cognitive and social development is taking place. Problems of young adolescence are compounded for disadvantaged students; reality for these students, both at home and at school, is very different from that of their peers. As these children progress through the school system they are in greatest risk of dropping out; nearly 60% of all Hispanic dropout students leave before completing 10th grade

(Carnegie Council, 1989). These problems will continue to escalate as the number of disadvantaged students in public schools continues to rise rapidly; by the year 2020 it is estimated that nearly half of all school-age children will be black or Hispanic (Carnegie Council, 1989).

The reality of school life is that disadvantaged students are disproportionately over-represented in low-ability groups (Goodlad, 1984; Oakes, 1985, 1990; Rosenbaum, 1976; Trimble, 1988). As the number of minority and disadvantaged students increases the need for pedagogical practices that respect, and take advantage of, student diversity intensifies. Levenson's (1979) study of sixth grade students succinctly noted the social advantages of the heterogeneous grouping method of organizing students: it provided a more democratic learning environment; no label was attached to the low-achievers; it helped to prepare for real-life situations by involving students with others whose race, culture, or ethnicity may be different from their own; students learned from their peers; teachers were less likely to assume that all students had the same instructional and social needs. By its very nature heterogeneous grouping prevents the segregation of students on the basis of socioeconomic status, ethnicity, or race (Oakes, 1985).

Ample evidence exists that schools are failing to meet the social needs of many young adolescents. The Carnegie Council (1989) has estimated that of the 28 million adolescents aged between 10 and 17 in the United States about half are at some risk of engaging in seriously damaging behaviors. Many young adolescents become involved with drugs and alcohol at an early age; 56 percent of 1987 graduating students began drinking in the sixth to ninth grades, the suicide rate for students aged between 10-14 years doubled during the years 1980-1985, sexual activity has dramatically increased for students under 16 years.

Middle schools "are potentially society's most powerful force to recapture millions of youth adrift, and help every young person thrive during adolescence. Yet all too often these schools exacerbate the problems of young adolescents" (Carnegie Council, p. 8). Rosenbaum's study (1976) provided an example of this view of schooling, commenting that at an important time of their lives too many of these students have their·inferiority communicated to them as authoritatively as the school can manage through the use of ability grouping. "In the balance of interactions between the individual and the environment out of which self-esteem grows, the environment is almost inevitably more powerful" (Beane, 1991, p. 27).

Effects of Ability Grouping on Instructional Practices

Simpson (1981) suggested that traditional organization of schools objectivates for students a shared reality consistent with the school's beliefs about individual ability. Other studies have supported the view that this shared reality is predicted upon organizational facets of classroom life, such as grouping arrangements, that provide experiences and meanings to classroom actors (Bossert, 1979; Goodlad, 1984; Oakes, 1985; Rosenholtz & Simpson, 1984; Rosenholtz & Wilson, 1980; Simpson, 1981). When students are grouped by ability a unidimensional evaluation results in a forced social comparison with peers. The negative consequences of this comparison may be feelings of intellectual inferiority, low aspirations, lack of motivation, interpersonal hostility, and competitiveness (Levine, 1983).

Traditional instructional and grouping practices may foster unidimensional classrooms where students

> all perform the same activities, are taught using whole group instruction, and are graded frequently. Given this uniform system of measuring achievement, imposed and measured by the classroom teacher, individual students have little opportunity to form more complex, and therefore more accurate opinions of themselves and their classmates. (Dawson, 1987, p. 355)

Rosenholtz and Wilson (1980) studied fifth and sixth grade classroom characteristics that produced variation in the degree of ability differentiation through teachers varying grouping practices and providing opportunities for multiple task differentiation and student autonomy. They found that classrooms that exhibited these characteristics provided a greater number of opportunities for students to demonstrate competence. Studies conducted on the effects of classroom organization and instruction on students' self-concept found that in multi-dimensional classes the structure of multi-task, and multi-group activities created an environment within which students had greater opportunities to form more complex, and therefore more accurate assessments of themselves and others. The homogeneous classrooms restricted opportunities for positive comparisons. (Bossert, 1979; Rosenholtz & Rosenholtz, 1981; Rosenholtz & Wilson, 1980; Simpson, 1981).

These studies were influenced by Berger and Luckman's (1967) theory of the social construction of reality, as was the current study, which responds to the need for "descriptive studies capable of contrasting teachers' and students' behaviors and perceptions in alternative grouping arrangements" (Slavin, 1987, p.321). The study is supported by Magoon's belief that the constructivist approach is most appropriate when "an account needs to be taken of external structural constraints that have a heavy influence on individual constructions" (Magoon, 1977, p. 665).

The learning environment and teacher behaviors associated with heterogeneous grouping were studied in 27 junior high schools by Evertson, Sanford, and Emmer (1981). Their findings indicated that extremely heterogeneous classes call for teachers who are skilled classroom managers and able to use a variety of teaching strategies. These teachers gave special attention and in-class assistance to low ability students, used peer-tutoring and some within-class grouping, used a variety of materials, considered individual student levels when giving assignments and grades, provided frequent academic feedback to all students, and maintained high levels of student accountability.

Shann (1990) pointed out that most studies of effective teaching behaviors have been carried out at the elementary level. A few have been conducted at high schools but little attention has been paid to middle schools. Her study revealed that middle school teachers who used more non-traditional methods, for example more small group instruction and less use of workbooks, had students with higher levels of achievement in reading, mathematics and problem solving; effective teachers were concerned with the social and personal, as well as the cognitive, development of students. Data taken from studies of secondary teachers support the view that learning gains are most related to classroom climate. "Affective teacher variables such as gaining student respect and forming good relationships with students appear particularly important" (Good, Biddle & Brophy, 1975).

Other studies indicated that effective teaching behavior is often determined by the way students are organized for instruction (Bossert, 1979; Dawson, 1987; Rosenholtz & Rosenholtz, 1981; Shann, 1990). Traditional organizational practices based on students' academic ability frequently created an environment that limited teachers' use of varying instructional strategies and forced them into an authoritative teaching mode of lecture, example, and seatwork. For early adolescents the most common method of organizing students is ability grouping. Grouping students in classes according to their achievement is widespread in middle schools (George, 1988).

Many teachers assumed that this is the most efficient method of organizing students, enabling teachers to adjust instruction to students' individual knowledge and skills. Yet studies reveal that teachers of ability grouped classes make little attempt to alter teaching strategies to accommodate students' individual differences; teachers use approximately 2% of instructional time in corrective feedback and the majority of teachers use the same methods as those used by their own teachers for all levels of ability (Goodlad, 1984; Oakes, 1985; Sirotnik, 1983).

Anderson and Barr (1989) have found that these traditional instructional arrangements are resistant to change. In their study of one district's three year attempt to change attitudes and practices related to grouping arrangements, they noted that teachers hold strong beliefs that ability grouping is beneficial for students. They have argued that educators' perceptions of appropriate instructional strategies must be changed. Teachers need training and more creative approaches to the curriculum are essential. Anything that makes heterogeneous grouping difficult must be eliminated.

Approximately 8,000 sixth-grade students were included in the Anderson and Barr study, which addressed how differing school organizational patterns affected academic learning of students with differing backgrounds and abilities. One school attempted to increase the number of students taking algebra. Seventy pre-algebra students were considered academically eligible for the move to algebra but because of scheduling difficulties only 20 were actual candidates. Curriculum differences were given as the reason for a final selection of five students for algebra. However, by the end of one semester of the two students who remained in the class only one was experiencing success. Once students are ability grouped, upward movement is virtually non-existent (Anderson & Barr, 1989).

Esposito (1973) noted that when classrooms were organized according to either a heterogeneous or homogeneous grouping principle the essential program conditions and consequent patterns of instruction did not differ. Ball's (1981) study also indicated that traditional instructional techniques of teacher-directed instruction and seatwork are used with students regardless of grouping practices. When school organization changed from homogeneous to heterogeneous grouping, teachers perceived the major advantage to be improved social relationships. Teachers' major concern was how to maintain academic standards, yet few altered their traditional instructional practices. Ball (1981) noted that the few changes that did occur in teaching methods during this first year of heterogeneous grouping were adaptive responses to the practical problems of teaching a wide range of abilities. These new techniques included individualized treatment in math, or within-class ability grouping, but often resulted in reinforcing differences in academic achievement. Those teachers who had been concerned with the egalitarian possibilities of heterogeneous grouping considered that these possibilities had been accomplished in the organizational creation of mixed-ability groups rather than the process and practice of teaching these groups (Ball, 1981).

Bossert's study (1979) demonstrated the importance of the effects of instructional practices on students. His study began in classrooms organized through ability grouping. This caused negative effects; teachers were forced into a role of instructional authority and cooperative behavior between students was inhibited. When the teachers changed their organizational structure and teaching style the social dynamics of the classroom changed also. Bossert pointed out that the structure of activities determined the shared experiences and possible interpretations of events and therefore shaped students and teachers perceptions of ability.

Many other researchers reported instructional differences in high-ability and low-ability classes (Cicourel & Kitsuse, 1963; Gamoran, 1987; Goodlad, 1984; Oakes, 1985). High-track teachers are often more enthusiastic, spend more time preparing for class and push students to work harder. Instruction in low-tracks often consists of worksheets and recitation-type methods that result in oversimplified, boring, repetitive lessons, often reflecting an overemphasis on control rather than educative processes. Sedlak et al., (1986) has suggested that this de-emphasis on academic learning has the primary goal of making the relationship between students and teachers less troublesome. It may be perceived as a bargain, tacitly negotiated between the two parties with perceived benefits for both; less academic demands are made on students and the reduction of student-teacher interactions lessen classroom tension.

The constructivist view suggests that as a result of interactions with the environment, students transform and organize their reality perceptions according to shared cognitive understandings; meaningful knowledge is constructed by members of a group in a highly interactive way. Therefore group heterogeneity may encourage diverse ways of learning. "Teachers and students are active constructors of the knowledge their share about each other. Performance is socially negotiated and the outcome of this negotiation depends in part upon the opportunities made available and constraints imposed by external conditions" (Rosenholtz & Rosenholtz, 1981, p. 133).

Some researchers consider a positive environment is one "where the distribution of opportunities to construct identities is large, individuals may select among a variety of performance options as the bases of social comparison and self-evaluation" (Rosenholtz & Rosenholtz, 1981, p.133). Heterogeneous classrooms provide greater opportunities to structure accurate perceptions of self. Inflexible homogeneous grouping arrangements influence students' self-perceptions and limit their opportunities to compare varying dimensions of ability. (Dawson, 1987; Goethals & Darley, 1987; Rosenholtz & Rosenholtz, 1981; Rosenholtz & Simpson, 1984; Rosenholtz & Wilson, 1980; Simpson, 1981). Too frequently students learn that good performance is taken by the teacher to mean high abilities and that this results in prestige and privilege. This value system is often echoed by students. The potential for students to adopt artificially low conceptions of their own abilities is great (Goethals & Darley, 1987).

Social comparison is significant during early adolescence. Piaget's fixed-stage theory of cognitive development was cited by Berndt (1982, p. 1450) to indicate that by early adolescence there is an analogous change in children's perceptions of equality; they become less selfish, realizing the importance of treating others as they would wish to be treated. Young adolescents recognize the importance of responding to individual differences that cause inequality (Berndt, 1982). Cognitive changes that occur during early adolescence influence the way students see and evaluate themselves and others. Cognitive characteristics of early adolescents have important implications for middle school educators in the creation of programs whose instructional techniques meet the emotional as well as the intellectual needs of young adolescents (Eichhorn, 1984).

Toepfer (1990) has suggested that organizational practices that provide multiple opportunities for determining student ability are especially important for young adolescents. He stressed the importance of providing teachers with training in various grouping programs that are more likely to meet the needs of early adolescents. Sykes (1986) has indicated that to be effective teachers require knowledge of the effects of differing instructional practices,

> But they also require knowledge of a more theoretical, abstract sort necessary both for developing a coherent approach to curriculum and instruction, and for interpreting the continuous flow of events in classroom and school. . . . Teachers must become students of their own students' learning in order to sensitively guide the

> learning process. . . . Teachers must become interpreters as well as nurturers of children's cognitive, social, and emotional development. (Sykes, 1986, p. 231)

Summary

This review of the literature examined the effects of ability grouping on students' achievement, and also on students' attitudes and self-concept. From this review of the literature it was evident that there is virtually no pre-existing research that contrasts teachers' and students' behaviors and perceptions in heterogeneous and homogeneous mathematics classes.

Symbolic interactionism guided much of the research reviewed in this chapter and provided the theoretical foundation for Berger and Luckmann's (1967) view of the social construction of reality. The purpose of the current study was to examine, in terms of constructivst theory, the opportunities to construct mathematical knowledge available to sixth-grade students when a teacher's classes are variously grouped for instruction. Constructivist theory supported the findings of the current study and also served as a means of organizing data.

CHAPTER 3
METHODOLOGY

The Research Perspective

The need for a study that looks at how various grouping arrangements shape learning opportunities has been well documented (Dawson, 1987; Gamoran, 1987; Gamoran & Berends, 1987; Oakes, 1990; Slavin, 1987). "In the absence of descriptive studies capable of contrasting teachers' and students' behaviors and perceptions we can only speculate about how the effects of various grouping plans may be produced. Such studies are clearly needed." (Slavin, 1987, p. 321)

Eisner (1991) has suggested that the growing interest in qualitative educational research "represents the beginning of a new way of thinking about the nature of knowledge and how it can be created" (p. 227). The present study, which was influenced by the constructivist perspective, supports Eisner's view that qualitative research "can not only broaden the ways in which we study schooling, but even more important, can expand our conception of human cognition and help us develop new forms of pedagogical practice" (p. 245).

The current study demonstrated the applicability of constructivist theory to educational research. The purpose of this study was to examine, in terms of constructivist theory, the opportunities to construct mathematical knowledge available to sixth-grade students when a teacher's classes were variously grouped for instruction. The review of the literature found much research on ability grouping; an abundance of quantitative studies that measured the effects on student achievement, for example those examined in the meta-analyses of C. Kulik and J. Kulik (1982, 1984, 1985), Henderson (1989), Noland and Taylor (1986) and Slavin (1987, 1990), and also many qualitative studies that examined the affective results of ability grouping (Ball, 1981; Goodlad, 1984; Hallinan & Sorensen, 1985; Oakes, 1985). There has, however, been a dearth of studies that explain how instructional practices are interpreted by students and their teachers within classes variously grouped for instruction.

Erickson (1986) suggested that the object of interpretive studies of this type is not to describe behavior but to describe action; the actions humans take on the grounds of their interpretations of the actions of others. He asserted that once those interpretations are made they are taken as real. For example, once a child has been assessed as belonging to a high or low ability group this means that two assumptions have been made: (a) that a condition or entity of high, or low, ability actually exists; and (b) that we can attribute it to that child. Constraints have now been placed on the child to learn and the teacher to teach.

> From an interpretative point of view, teacher effectiveness is a matter of the nature of the social organization of classroom life--what we have called the enacted curriculum--whose condition is largely, but not exclusively, the responsibility of the teacher as instructional leader. (Erickson, 1986, p. 133)

A major strength of the current study was that all classes had the same, very capable, mathematics teacher. It was therefore possible to relate the differences that occurred in instructional practices, and in social and mathematical interactions, to the effects of various grouping practices.

Entry

In the spring of the year preceding the study the researcher had shared recent experiences of heterogeneous mathematics classes with participants at a local conference on middle schools. Administrators from the school included in the study expressed interest in these experiences and indicated that their school was moving away from the traditional method of grouping students according to ability. Prior to the end of that school year, the researcher contacted a sixth-grade mathematics teacher who would be using a variety of organizational strategies during the following year. This teacher, and the school administrators, enthusiastically welcomed the opportunity for research to be conducted at the school on the effects of organizational and instructional practices on students. One administrator remarked that having an outside observer could prove to be advantageous in supporting the results of the various grouping arrangement effects.

After making arrangements to conduct the study at the school, the researcher later accepted an offer to become one of the eighth-grade mathematics teachers at the school. This did not affect the study adversely as the size and organization of the school were such that virtually no contact took place between teachers and students from other grades. There were many benefits to the study as the researcher was able to conduct research more frequently and had easier access to key informants.

The Setting

The school is located in the more affluent area of a southern university city of approximately 90,000 residents. Built in the 1960's, with an open-plan design, the school is overcrowded and in need of major renovation. During the year of the study it housed more than 1100 students in grades six, seven, and eight.

The school has long held a reputation for being strong in the academic subjects, and for years there were many ability levels in all three grades. Teachers tend to remain at the school for many years as the majority of the student body comes from middle-class homes that encourage academic involvement. Approximately 22% of the school population consists of black students bused from the poorer eastern areas of the city. A small minority of white students live in trailer parks located along the interstate highway.

Given this relatively diverse student body, and an innovative administrative team, efforts were made in the last two years to reduce the homogeneity of academic classes. This caused concern, particularly among parents of those students in the advanced groups. Administrators and teachers set a goal that within the next two years there will be one common mathematics course at the sixth- and seven-grade culminating in an algebra course for all eighth-grade students. This approach has been phased in gradually, with the sixth-grade mathematics classes providing the foundations.

There are two sixth-grade mathematics teachers at this school. Sixth-grade students in the past have been placed in one of four ability groups, compensatory, basic, enriched and advanced. During the year of the study students in Mr. Abbott's mathematics classes were organized in one of three ways: (a) they were placed in advanced classes, where all students achieved above the 92nd percentile, as measured on the mathematics section of the fifth-grade California Achievement Test; or (b) they were placed in a pilot heterogenous group of students whose achievement scores were representative of the entire sixth-grade population; or (c) they were randomly assigned to basic heterogeneous classes that were minus the advanced students. (All proper names were changed in this study to protect the identity of informants.)

Mr. Abbott taught five mathematics classes each day: two advanced classes, two basic heterogeneous classes, and the heterogeneous class. This last group was, for the purposes of this study, named the pilot group, as the administrators intended this group to lead the way to total heterogeneous grouping school-wide. Mr. Abbott was a white, middle-class, man in his early fifties and was highly regarded as a teacher of mathematics by parents, students, teachers, and administrators.

Research Procedures

Adhering to the theoretical tenets of constructivism and symbolic-interactionism, the researcher adopted a participant-observer role during the course of the study. Spradley (1980) noted that this role entails coming to the social situation with two purposes: (a) to engage in appropriate activities; and (b) to observe the activities, people, and physical aspects of the situation. Thus the researcher becomes the research instrument, and the research questions guide the collection of data. In the current study the constructivist perspective informed the following questions that guided the research:

1. How does the delivery of curriculum and instruction differ in the heterogeneous and homogeneous mathematics classes observed?
2. How do social and mathematical interactions differ in the classes observed?
3. What constitutes meaningful mathematical learning in the various classes?

The intensive study of heterogeneous and homogeneous classrooms through the use of qualitative methods of observation, in-depth interviewing, and videotaping, adheres to the tenets of Berger and Luckmann's (1967) theory of the social construction of reality. Additional triangulation was provided by data from attitude surveys and achievement tests.

Data from Observations

Merriam (1988) stated that there are several stances that can be assumed while collecting data as an observer. They range from complete participation, where the researcher becomes a member of the group, to complete observation, where the researcher is hidden from the group. In reality researchers usually mix observation with participation.

Classroom observations averaged four each week for the majority of the school year, and lasted for entire class periods. The number of observations reduced to two each week in May, as the data became increasingly repetitive. Classes were observed for more than 120 hours. On many occasions the various groups were observed on the same day as this facilitated comparison between the mathematics classes. During the early part of the study the researcher was a passive participant observer so that both teacher and students became familiar with her presence in the classroom. Later the researcher's level of involvement in the social interactions of students increased. The extent to which this participant-observer affected the social situation was minimal. A detailed record was kept of what was observed and experienced. Both objective observations and subjective feelings were recorded on the spot. Further details were added as soon afterwards as possible.

Data from Audiotapes and Videotapes

The written record was expanded through the use of audiotapes and videotapes. These tapes augmented the fieldnotes. Videotaping classroom practices was especially appropriate for this study as Mr. Abbott was comfortable with the practice, and had already intended to use this device to assess his performance.

Videotaping was used on ten occasions throughout the year to record the happenings within the various mathematics classes. Recording the experiences of mathematics students and their teacher provided opportunities for comparisons between the different mathematics groups. Videotaping was also used to collect data on four occasions when the researcher was not able to be present. On those occasions the camera was set up to record the actions of the teacher and approximately half the class; when the researcher was present with the camera interactions between the teacher and all students were recorded. Eisner (1991) suggested that videotaping has advantages for the collection of data. For example, it provides opportunities to compare the teacher as he teaches the same topic to students in various grouping arrangements. It also helps the researcher and the teacher to be introspective about the classroom experiences. This stimulated recall, which may involve using either audiotapes, or videotapes, provides opportunities that "serve(s) as a stimulus to elict the teacher's thought processes" (Shavelson et al., 1986, p. 80).

Interview Data

The constructivist perspective influenced the interviews as well as the research questions. Interview questions centered around the consideration of the nature of mathematics classes; they were related to the meaning of mathematical knowledge and the implications for pedagogical practice.

Many informal conversations took place between the researcher and teacher, and also with students. An interview however, is a conversation with a purpose (Merriam, 1988). Interviews varied from the conversational format to the highly structured interview where questions were well-organized ahead of time. There were twenty-four formal interviews during the course of the study; three of these were formal interviews with the teacher.

Interviews were also conducted on a regular basis with students who were representative of their class (see appendix for interview guides). As the students became known to the researcher, a purposive, rather than random, sample of fourteen students was interviewed throughout the school year. Borg and Gall (1989) have suggested that it may sometimes prove more informative to select certain subjects for interviewing. "Questions such as 'Why does this subject repond differently?' and 'What implications do these differences have in developing theory?' can be very productive in advancing research knowledge." (p. 386)

Students interviewed for the study included six male and eight female students of all abilities, who were representative of the racial and socio-economic groups present in the school. Four students in the sample came from the advanced classes, five came from the basic classes and five from the heterogeneous pilot class. A general description of these students' mathematical ability is that six students were above average, four were average, and four were below average. Three of the students were Afro-American and three were of Asian origin. After the recorded interviews were transcribed and analyzed, some students were interviewed for a second time in order to clarify and expand original answers.

As well as being asked questions that centered around the nature of the mathematics class, students were also questioned in an attempt to determine the effects of school organization on students' social interactions. Students were asked such questions as: Which three people would you choose to work with? Why? Who do you choose to eat lunch with? and, Have you made any new friends in this class?

Interviews were also conducted with key-informants, such as school administrators, and educators from the College of Education who spent time in Mr. Abbott's classroom for several weeks of the year. These key-informants had special knowledge and insights valuable to the researcher.

Data from Achievement Tests and Surveys

The mathematics perceptions of all students in the various mathematics classes were assessed in a variety of ways. Included in this assessment were test scores, responses to questions that determine attitudes, and checklists of adjectives. This triangulation of methodology, the use of several different kinds of data-collection instruments to study the same issue, contributes to confidence in the research findings (Borg & Gall, 1989).

A panel of six educational researchers from the College of Education, helped code students' responses to questions that attempt to assess their attitudes toward mathematics, such as: What do you like best about your math class? Is math one of your favorite class? Why? Why not? Do you worry when you take a math test? Why? Why not? These responses were typed and written in standard English so that the panel remained unbiased by students' misspelling or use of local dialect.

A variety of methods was used in an attempt to determine the attitudes toward mathematics of students in various mathematics classes. All students included in the study were given checklists of adjectives, such as "boring", "fun", "easy", and "exciting", that described their classes and how they felt about mathematics and their mathematics ability. The results of this triangulation from test scores, questions, and checklists, enabled the researcher to obtain a multiple perspective on students' attitudes toward mathematics. Data was also collected through unobtrusive measures, such as an examination of the teacher's lesson plans, grade book, and students' achievement levels on homework and tests.

Students' scores on the mathematics achievement test that they took in September were compared with their May scores on the same mathematics test. Descriptions of scores and change of scores is embedded in the discussion of the findings.

Data Analysis

The opportunity to use multiple methods of data collection was a major strength of this study. Although each method had advantages and disadvantages, the synergetic effect of this triangulation proved powerful. Ongoing analysis of data took place throughout the course of the study as categories emerged from the transcribed data which further focused the perceptions of the researcher. Merriam (1988) warned that without ongoing analysis, data become unfocused, repetitious, and overwhelming.

Merriam (1988) has summarized the components of data analysis as first the collection of raw data from which categories, themes, or other taxononmic classes are developed that interpret the data. During the course of this study, data were classified into categories, as answers were obtained to such questions as: Who is in the group or scene? What are these students' identities and how did they become members of the group? What is happening here? What are these students doing and saying to each other? How are activities organized and what resources are used? What is the nature of these students' interactions? What roles are evident in these interactions? What kind of learning is taking place?

These categories and their relationships were then linked by a tentative hypothesis that moved toward the development of a theory to explain the meaning of the data.

Merriam (1988) stated that moving from the basic level of data analysis to a more abstract level, one that involves using concepts to describe phenomena, is a step toward developing a theory that "explains some aspect of educational practice and allows one to draw inferences about future activity" (pp. 140, 141).

These methods of data analysis were particularly appropriate given the constructivist perspective taken by the researcher. "The scientific goal . . . is . . . the careful description and validation of a patterned set of interrelationships." (Magoon, 1977, p. 668)

Methodological Issues

Assessing qualitative research nessitates the use of different criteria than is used in traditional research (Merriam, 1988). Borg and Gall (1989) listed mistakes sometimes made by qualitative researchers: (a) using methods for which they were not trained; (b) depending entirely on participant observation; (c) allowing preconceived ideas to influence their observations; and (d) observing for too short a time. The researcher took the following measures that she considered necessary to produce a high level of validity in the fieldwork and analysis:

1. The researcher was a capable primary instrument of research as she had successfully completed four courses in qualitative research, including ethnography.

2. The researcher used multiple methods for the collection of data; these included, but were not restricted to, observation, videotaping, and in-depth interviewing.

3. The researcher acknowledged a constructivist perspective. Background experiences as mathematics teacher and graduate student were taken into consideration as potential sources of bias as the data were collected and analyzed.

4. The researcher conducted research over the entire school year. On average, classes were observed for four 45 minute periods each week. Informants were interviewed throughout the school year. Adult interviews averaged one hour in length, student interviews averaged 35 minutes. The total number of hours spent in direct contact with the teacher and students exceeded 140 hours. Another 12 hours of classroom observation were videotaped.

Validity and Reliability

A qualitative study is concerned with interpreting reality by producing valid and reliable data in an ethical manner. Lincoln and Guba (1985) stated that reality is a multiple set of mental constructions and that these constructions are valid if they "are credible to the constructors of the original multiple realities" (p.296). In this study the teacher and students had opportunities during the interviews to comment on and verify interpretations and observations made by the researcher and other informants. When reality is viewed in this way, internal validity is a definite strength of the qualitative study (Magoon,1977; Merriam, 1988).

Triangulation, using multiple sources of data, also ensures internal validity. The findings from this study interpreted data from observations, interviews, videotaping, questionnaires, test scores, and unobtrusive measures, that were collected through one school year.

The traditional meaning of reliability, which demands that other researchers get the same results, seems inappropriate for qualitative research (Merriam, 1988). Instead, one would wish others to agree that, given the data collected, the results are dependable and consistent (Lincoln & Guba, 1985).

This study was conducted to gain in-depth understanding of what occurs in particular classrooms, not in order to obtain general knowledge of many classrooms. Erickson (1986) considered it inappropriate to attempt to generalize the results of interpretive research. Interpretative researchers attend to "the details of the concrete case in hand. Thus the primary concern of interpretative research is particularizability, rather than generalizability" (p. 130). The generalizability of this study depends on the reader and the situation to which the reader wishes to apply the results. Therefore, it was necessary to provide thick, rich, description "so that anyone interested in transferability has a base of information appropriate to the judgment" (Lincoln & Guba, 1985, p. 124, 125).

Careful measures were taken to protect the anonymity of the subjects. Permission to interview students was granted by the University of Florida Institutional Review Board. The informed consent of parents and students was obtained before any interviews took place. All names were changed and any identifying characteristics altered. Audio- and videotapes were erased as soon as all useful data were extracted. Every effort was made to record unbiased, accurate, data.

CHAPTER 4
EFFECTS OF VARIOUS GROUPING PRACTICES ON SIXTH-GRADE MATHEMATICS STUDENTS

Introduction

The purpose of this study was to examine, in terms of constructivist theory, the opportunities to construct mathematical knowledge available to sixth-grade students when a teacher's classes were variously grouped for instruction. This intensive study of homogeneous and heterogeneous classrooms took place over the course of one school year. Three research questions guided the study:

1. How does the delivery of curriculum and instruction differ in the heterogeneous and homogeneous mathematics classes observed?
2. How do social and mathematical interactions differ in the mathematics classes observed?
3. What constitutes meaningful mathematical learning in the various mathematics classes observed?

Qualitative methods of observation and in-depth interviewing were used for the study. These methods adhered to the tenets of Berger and Luckmann's (1967) theory of the social construction of reality. Twenty-four formal interviews took place during the course of the study.

Observations in the various classes were conducted an average of four times each week during the majority of the school year. By May the number of observations had reduced to two observations each week as the data became repetitive. Classes were observed for a total of 120 hours.

All general observations were made in the context of Mr. Abbott's mathematics classes. Structured observations and interviews were conducted in a small room adjacent to the mathematics classroom. Observation data were either recorded as handwritten notes which were later typed and expanded, or recorded on videotapes and then transcribed. Interview data were recorded on audio-tape and later transcribed. Other research methods used in this study included data from attitude surveys and achievement test scores. These data provided additional triangulation.

The constructivist methods of data collection used for this study made it possible to observe the teacher and his students as active constructors of mathematical knowledge and to determine the constraints that limited these constructions. According to constructivist theorists, meaningful mathematical learning occurs only when learners construct their own knowledge. For example, this may occur when students use manipulatives to construct mathematical representations, when they use interactive videos or computer programs, or when they interact mathematically in cooperative groups. Constructivists believe that students construct their own mathematical knowledge when they become involved in complex processes that require them to think creatively and analytically about problems.

Analysis of the data collected during the first months of the school year resulted in the emergence of various descriptive categories. Later, when the school year ended, all the various kinds of data that had been collected were subject to further detailed analysis in terms of constructivist theory. This resulted in the development of several major categories of findings that converged under one major heading: Opportunities to actively construct knowledge available to students grouped heterogeneously, or homogeneously, for mathematics instruction. When the data were organized in order to reveal and explain how classes differed in time afforded for exploring new and different ways of constructing mathematical knowledge, evidence was found that some classes were more frequently engaged in constructivist activities than others.

Findings will be discussed under three major headings that reference the research questions that guided the study:

1. Effects of labeling classes on instructional practices
2. The importance of social and mathematical interactions in the learning process
3. Opportunities available to students in variously grouped classes for the active construction of

mathematical knowledge

Prior to further discussion of these findings, an introduction to the informants in the study, and to the various mathematics classes, may help place the findings in context.

Setting for the Study

By providing detailed information on the informants in this study, and by providing opportunities to vicariously experience a typical day in the individual classrooms, it is possible to determine how the reality of life in the various classrooms was a social construction that resulted from interactions with the environment.

Mr. Abbott, the Sixth-grade Mathematics Teacher

Mr. Abbott was a friendly, even-tempered, white male in his early fifties. He served as a weapons training officer in the military for three years, later becoming a language arts teacher and then a teacher of mathematics. His enthusiasm for the teaching profession was very evident and he was very well liked by his students. At the beginning of each class period Mr. Abbott usually positioned himself at the entrance to the classroom, speaking and joking with individual students as they entered. He taught mathematics to approximately 160 sixth-grade students each day during five class periods. The schedule rotated every 12 weeks so that students were not locked into the first, or last, period of the day for mathematics during the whole school year. Mr. Abbott taught two advanced classes, two basic classes and one heterogeneous class, which for the purpose of this study is called the pilot class. He had a double planning period each day and also had a homeroom class.

During the course of the year several adults worked with Mr. Abbott in the mathematics classroom. One, a professor of education, was experiencing the middle school during a sabbatical year. He spent each morning observing in Mr. Abbott's room during the first three months of school. He was followed by Ms. Dee, an intern from the College of Education, who spent approximately sixteen weeks with Mr. Abbott. Also, a parent volunteer regularly assisted in the classroom, working with individuals and helping with paper work.

Mr. Abbott's Sixth-grade Mathematics Classroom

Although the school was built in the 1970's with an open plan design, individual classrooms were later constructed through the installation of lightweight metal walls. Mr. Abbott's classroom was situated halfway along the sixth-grade hall. Classrooms had no doors and, as in the case of Mr. Abbott's room, often interconnected with other classrooms. A drinking fountain was located directly opposite the entrance to Mr. Abbott's room. Mr. Abbott used a strong, clear, voice to address students and his classes were less disturbed than others by activities in neighboring rooms or by those using the halls during classtime.

Mr. Abbott's classroom was a pleasant and well-organized room with no windows. It contained nine rectangular tables that each accommodated four students. There were three tables placed along the back wall of the classroom and two others close to the teacher's table which was directly in front of the chalkboard. Two tables were positioned along each of the two side walls. Under this arrangement twenty students sat sideways to the teacher and the chalkboard. Eight students at the side tables faced front, while eight other students at the four side tables had their backs to the chalkboard and the teacher's table. The side table nearest the hallway was not used during the pilot and advanced classes. Filing cabinets and bookshelves containing class sets of mathematics books and students' folders occupied the corners of the room.

On one wall of the classroom, Mr. Abbott had posted photographs of his 23 homeroom students. On other walls were many attractive photographs of horses and posters of hot-air balloons. These were colorful and had been laminated. Computer-made banners read "Don't forget your homework", "The properties of addition and multiplication", "Factor times factor equals product", and "Success is the intelligent use of our mistakes". When the classes were working on cooperative learning and problem solving later in the year, the banners were changed to "We instead of me", "Thirty heads are better than one", and "Strategies for problem solving with calculators".

Mr. Abbott was well-organized. Each day as the students from the three types of classes entered the room they saw their agenda for the day written on the chalkboard under three headings: Advanced, Advanced P (pilot), and Basic. The agenda listed the page number and problems that each of the three types of classes would be working on that day. It was apparent to students, on a daily basis, that the classes were following very different curricula. The weekly agendas for the three different groups were computer printed and placed in folders so that if students were absent they had easy access to the work that they had to make up.

In order to increase the learning time in each mathematics class Mr. Abbott instituted a point system that rewarded students who settled down quickly at the beginning of each period. When all students at a table were seated with their materials ready for the day's work, Mr. Abbott would place small reward cards on their table which were converted into points and recorded on a posted chart. These were totaled at the end of the week. The table with the most points was rewarded on Fridays with lollipops and candy. Students were enthusiastic about earning points and therefore urged members of their group to hurry and settle down. This points system was also used to reward groups of students, mostly in the basic classes, who worked quietly during the period. The following descriptions of the three types of classes will help place the findings in context by providing an accurate representation of life in the various mathematics classes.

The Basic Mathematics Classes

Mr. Abbott taught two basic classes. One contained 34 students; 17 boys and 17 girls. There were 14 minority students in this class, 10 of whom were Afro-Americans. The other basic math class contained 31 students; 17 girls and 14 boys. There were 6 minority students in this class, 5 of whom were Afro-American. Mrs. Brook, another sixth-grade teacher, taught five other basic mathematics classes where the number of minority students in each class averaged 32%.

All students not placed in either of the two advanced classes or the pilot class were divided between the basic classes. In Mr. Abbott's basic classes, students' mathematics scores from the fifth-grade California Achievement Test ranged from the 5th percentile to the 97th percentile. Six students in Mr. Abbott's basic mathematics classes scored above the 89th percentile on the fifth-grade mathematics section of the California Achievement Test. The approximate ratio of high achieving students scoring above the 80th percentile to those scoring below the 80th percentile was one to four.

A Typical Day in the Basic Mathematics Class

Mr. Abbott stood at the entrance to the classroom interacting with students as they entered the room. He called to a wiry sandy-haired boy, "Hey Joe, what was the score?" Joe gave him a favorable response and the two of them slapped hands in a 'high five' salute. Joe, flushed with pleasure as he made his way to his seat, answered questions about the game from other boys. Several other students greeted Mr. Abbott and he responded and joked with many of them as they entered. "Hey Mike, either your hat or your head comes off, son!" Mike, a tall black boy, grinned and removed the hat.

The bell signaled the beginning of class, Mr. Abbott moved to his table and picked up his pile of reward cards and late students scurried to their seats. Students took out books, papers, and pencils, and urged the slower members of the table to hurry as Mr. Abbott walked around the room giving four or five cards to those tables that were ready first. "Thank you, gentlemen," he said to two boys who were continuing a heated discussion about baseball cards. "Okay, we're going to check the homework and then get into today's assignment." As Mr. Abbott prepared the class for the day's work, two boys went to sharpen pencils at the electric sharpener and Latasha called to Jermeka, at the next table, for a sheet of paper. Erika, a large black girl, was searching through her bag for supplies and talking to the white girl next to her as she did so, but most students became quiet and Mr. Abbott stood behind his table and continued. "This is easy work. I think we'll have time to complete it in class, so you won't have any homework." This did not appear to be a great incentive; one boy gave three slow handclaps.

The teacher noticed two boys talking. "Calvin, Billy, it's my turn now. Turn to the top of page 123. We're going to do them all. They're so simple." Most students opened their mathematics books as Mr. Abbott placed a large decimal point on the chalkboard saying as he did so, "Last night you had to give the place value of decimal numbers. This is our decimal point." Two white girls at a side table turned in their chairs to face the board. Barry, a large boy who has been retained in the sixth-grade for a second year, sat at a side table with his back to the board and was concentrating on removing an eraser from the end of a pencil. Mick, a chubby Hispanic boy, sat opposite to Barry and was facing Mr. Abbott but his book was closed and he watched with interest as the Barry's eraser broke into several pieces. Mr. Abbott called on students to answer questions as he read from the book. "We have a three in the 10,000's place. Amanda, where shall we put that?" Amanda told him to place it five places to the left of the decimal point. "Does everyone see how she got that?" asked Mr. Abbott, as he wrote the three in the correct position on the board; there was no reply. Mr. Abbott told the class that if they had an incorrect answer on their paper they could change it and he reminded them that this was what would be on their test the next day. "We have a four in the 100's place. Where shall I put that?" Several students had lost interest. They yawned, stretched, and Anne put her head on the desk. Samantha was reading a book and Joel, a bespectacled white boy, was drawing. Only two students raised their hands.

Mr. Abbott attempted to rouse the class. "Hey, guys, come on, let's get moving. We can do better than this." Barry quietly flicked a piece of eraser at Brian, who was seated at the next table, as Mr. Abbott asked Mick the next question. "We have a five in the tenths place, Mick. Where shall I put that?" Mick reached for his textbook, quickly glancing at his neighbor's book to see the page number, as seven students raised their hands . Mr. Abbott waited as Mick shuffled the pages, saying, "That's okay. We're on page 123." Mick found the page but made an inaccurate guess at the answer. Mr. Abbott moved on to Sara, who was waving her hand in the air, and he wrote her answer in the correct place on the board. This process continued until the number 2,038,476.501 was written on the board. Mr. Abbott then called on Myneera, a chubby Asian girl, to read the answer aloud.

For the next 20 minutes Mr. Abbott continued to work out solutions to the previous night's homework. Ben yawned loudly and propped his chin on his hand. Samantha carried on reading and Joel continued drawing. Latasha picked up the blue, plastic, bathroom pass, disappeared into the hallway, and was gone four minutes. Brian, noticing her return, was prompted to get the boys' bathroom pass. Latasha handed over the pass to Jermeka just as Becky began to rise to her feet. Other students concentrated on writing in answers, or making changes to their papers as the answers were given; five students had no papers in front of them. At the end of this time Mr. Abbott asked, "Does anyone have any questions?" There was no response and so one person from each group brought the papers up to Mr. Abbott's table. "Now I'm going to give you four minutes to complete this worksheet," Mr. Abbott continued, as he walked around placing four papers at each table. "It's just like you were doing last night for homework but now I want you to write in the words as well as the numbers. If everyone at your table gets them all right you can earn extra points. I'm coming round to give points to people who are working quietly." Myneera made a face at Yolanda, who was sitting opposite to her and Yolanda rolled her eyes. Ryan, a white boy who had been in a compensatory mathematics program the previous year and who now always sat at one end of the teacher's table, dropped his head theatrically onto the table.

Mr. Abbott walked around the room distributing reward cards and quietly answering questions as students worked.

The room was quiet. Some students were sneaking looks at their neighbors' papers, either to check their answers or their spellings. Three or four students were blatantly copying solutions from others at their table. After four minutes several students had finished. Samantha resumed reading her book and Myneera also reached for a reading book. Megan and Yolanda began drawing. Brian leaned back and rested his head on the wall. Students began to talk quietly among themselves. After six minutes Mr. Abbott asked who had not finished and several hands were raised so he gave them another five minutes. Mr. Abbott then prepared to give the class the solution. "Okay. This is the answer you should have." However, Yolanda was eager to give the answer herself and she waved her hand in the air. So Mr. Abbott called on her instead and she correctly said 3 million, 784 thousand, 612 and 524 ten-thousandths.

"Put this paper in your folder," said Mr. Abbott, "and use it to review from. Remember this is what will be on your test tomorrow. No surprises. Now I'm going to give you another worksheet for tonight's homework." Students talked and called to friends at other tables as they put the work away in folders. Amanda admired Megan's drawing and Mick and Barry were having a 'thumb war' as Mr. Abbott placed the next worksheet on their table. Several students, including Myneera, Yolanda and Megan, started work immediately the worksheet was given to them. "Okay," said

Mr. Abbott. "This is the same thing but it's like a crossword. You need to change the words into numbers and place them in the crossword puzzle so they all fit. As some students appeared to be looking at the paper in bewilderment, Mr. Abbott said, "Let's look at the first problem together. The number is 534. Where could it go?" Two boys at one of the back tables had already folded their worksheets and had placed them in their backpacks, and Ericka was folding her paper meticulously, but most students looked at the worksheet. No one responded so Mr. Abbott continued. "This number has no decimal point so there are two places it could go, either at the top or the bottom. I'll give you a clue, it goes at the bottom." The noise level, which had been minimal throughout the period increased during the last minutes of class, as students packed their backpacks and put their folders away, while a few worked on their homework. Myneera smiled triumphantly, completing the worksheet as Mr. Abbott said, "Okay. It's almost time for the bell." Two white girls moved up to the teacher's table and chatted with Mr. Abbott as he returned spare worksheets to the filing cabinet. Other students moved across the room to chat with friends. As the bell sounded Mr. Abbott called out above the noise. "You did well. See you tomorrow."

The Pilot Class

School administrators first decided to move towards heterogeneously grouping students in mathematics by offering a pilot program at the sixth-grade level. Prior to the new school year, parents were invited to an evening meeting at the school to hear about the program. After the meeting, which administrators reported as the largest parent meeting ever held at the school, more parents signed their children for the program than could be accommodated. Sufficient parents of all races attended the meeting so that, from these volunteers, a group was selected that administrators reported as being "a stratified sample that represents the sixth-grade population."

Thirty-two students were placed in the pilot class; 18 girls and 14 boys. There were five minority students in the class, four of whom were Afro-American. Students' mathematics test scores, from the fifth-grade California Achievement Test, ranged from the 35th percentile to the 99th percentile. Both the mean and median mathematics score of the entire sixth-grade population was at the 71st percentile. In the pilot class, five students had mathematics scores that were less than the 50th percentile, eight students had scores between the 50th and the 75th percentile, nine students had scores between the 75th and 89th percentile, and ten students had mathematics scores at the 90th percentile or above. The ratio of high achieving students scoring above the 80th percentile to those scoring below the 80th percentile was approximately three to two. Parents chose to place their students in this pilot program; several parents of these students were either teachers or associated with education in some way.

A Typical Day in the Pilot Class

Students piled into the classroom, teasing and joking with each other as they made their way to their seats. Mr. Abbott made his way to his table calling out to Mary, "Good to have you back." Tom became the center of attention of a small group of boys as he tossed magnets onto the metallic classroom wall. Two boys followed Mr. Abbott as he gave out reward cards and were laughing with him as the bell sounded. Beth leaned across the table to Anna saying, "What did you get for number 18?" Several other students were comparing homework and chatting as they prepared for the lesson. Denny was copying answers from Natalie's paper. This class was lively and the noise level caused Mr. Abbott to whistle to attract students' attention. "Ladies and Gentlemen, let's get started!"

This period began with a reminder about re-taking last week's test. "Several of you have asked to retake the test and others have already done so. You really ought to do it if you got lower than an 85%. If you raise your hand I'll give you a copy now and you can do it for homework." Mr. Abbott handed out about six tests. "Okay. Let's grade the homework." He read the answers aloud as students graded their papers. At each of the first answers Denny said "Yes!" loudly and with great enthusiasm to indicate to Mr. Abbott that his answers were correct. Mr. Abbott raised his head from the book. "We really don't need a cheering section," he said mildly. Denny grinned and several students giggled. When the answer to number 18 was reached, Beth asked Mr. Abbott to explain the problem. "How come I got ten for my answer," she said. A few students began talking quietly to others at their table, but most watched as Mr. Abbott worked through the problem on the board. When the homework was graded one person from each table collected it and handed it in.

Now the period moved into teacher directed time. "Okay. Scott, Denny, Carrie, thank you for getting quiet! I want to get started on equations and inequalities." The next 20 minutes were used by Mr. Abbott in an interactive question and answer session. At first some of the slower mathematics students were inattentive. Denny was showing Natalie a baseball card, James was looking around the room and Gisella was trying to talk with Vanessa. Vanessa, ignoring her, turned to the board and concentrated on Mr. Abbott. Gradually the class settled and became on task as Mr. Abbott called on students to answer. "Six times three is eighteen. Natalie, give me a related statement." "Do it backwards, " said Natalie ambiguously. "Does it help if I write it up?" asked Mr. Abbott writing the equation on the board. As Natalie hesitated Jason blurted out, "You've got to change from multiply to divide." "Who can give me the related statement then," asked Mr. Abbott. Denny half raised his hand and then said "No, forget it." "I don't want to forget it," said Mr. Abbott. "What would the related statement be, Denny?" "Eighteen divide by six is three," said Denny. Vanessa waved her arm in the air saying, "It could also be eighteen divide by three is six."

All students were on task as they practiced giving related statements. Many volunteered answers but Mr. Abbott also called on those who did not volunteer. "So if we have the equation five times a number is thirty," continued Mr. Abbott writing 5X = 30 on the board, "how would we solve it? Jason?" "Divide by five," Jason replied. "So if you're doing the problem would you put six as the answer?" Vanessa asked. "Yes," replied Mr. Abbott, "but I'm not bothered about the answer at the moment. I want you to understand the process. Think back to when we did addition and subtraction in equations. Does the same thing apply, Carrie?" He wrote t + 6 = 18 on the board. "Yes," said Carrie, "subtract six." Mr. Abbott wrote t + 6 - 6 = 18 - 6 saying, "We need to balance the equation. What am I left with, Scott?" asked Mr. Abbott. Scott was beaten to an answer by Anna. "Twelve," she said. Scott looked at her and said, "Why isn't it 24?" Anna leaned across and swiftly wrote on his paper. "If we have $t - 6 = 18$ do we have to balance it?" Sarah asked. Mr. Abbott demonstrated by working the equation on the board. "Oh, is this what we always do?" asked Vanessa. "This is algebra," Mr. Abbott replied. Beth grimaced at Anna. The class looked impressed.

Denny and James had been fooling around off task for the last couple of minutes, and were becoming noisy. "He's stealing my stuff," said James smiling, as Mr. Abbott looked their way. Mr. Abbott swiftly moved Denny's seat so that it was further from James and continued with the lesson. During the interruption Gisella took the bathroom pass and disappeared into the hallway. Mr. Abbott now progressed to inequalities writing $3W < 42$ on the board and reading it aloud as he did so. He asked James, "Three goes into 42 how many times?" "Fourteen," answered James emphatically. "What numbers are less than 14?" Mr. Abbott asked. Hands shot up and students called out possible solutions as they rushed to answer. "Did we change how to solve it?" asked Mr. Abbott. No, came back the response. David, a quiet thoughtful black boy, raised a question about a problem in the book. "Would number seven be any number less than four?" he asked. "Let's all look at number seven," said Mr. Abbott. "How many people agree with David? TJ?" "Yes," said TJ. "Okay," said Mr. Abbott. "Let's see how well you can do this assignment." Denny took the boys' bathroom pass and disappeared.

As students worked through the last ten minutes of the period a few worked quietly alone but most interacted with others at their table as they did the problems. "What did you get for number 20?" asked one girl. "Fifteen," another responded. "How come I got three?" said the first girl. "That's what I got too," a boy interjected. He glanced over at the second girl's paper and pointing to the problem said, "See, you forgot to divide." At another table Jason asked Kelly, "What did you get for number twelve?" She turned her paper round to show him. "You got this mixed up," he said. "No, I didn't," said Kelly indignantly. "Yes, you did," said Jason. "Look," and the two students put their heads together across the table. Denny returned from the bathroom after several minutes and asked Natalie, a quiet black girl, how to do number four. Chip and Scott worked as a pair doing each problem together. Chip called out, "Mr. Abbott, we're stuck on number sixteen." Throughout this time Mr. Abbott walked around the room helping students. "Okay, class," said Mr. Abbott. "It's about time to wrap up." "Shoot," said Vanessa quietly, "I'm not finished." "Finish this page for homework. Let's clear away," said Mr. Abbott as the bell rang.

The Advanced Mathematics Classes

Mr. Abbott taught all the advanced sixth-grade mathematics students. Group membership was determined by a score at the 9th stanine on the mathematics section of the fifth-grade California Achievement Test. The lowest mathematics scores for students in the two advanced classes were at the 93rd percentile.

One advanced class contained 33 students; 19 girls and 14 boys. Of these 33 students 7 were of Asian origin. The other advanced class contained 31 white students; 13 girls and 18 boys. There was one Asian boy in this class. There were no black students in either of the advanced classes.

A Typical Day in the Advanced Mathematics Classes

Mr. Abbott moved from the classroom entrance toward the teacher's table as students entered the room, joking and talking as he did so. Anne, a tall girl with a dark ponytail, stood by her chair, her backpack still over her shoulder, reading the agenda for the day that was written on the chalkboard, under the heading "Advanced". Nick, a short skinny dark-haired boy, moved to the adjoining table and began reading the agenda aloud. Two boys, with an open mathematics book, questioned Mr. Abbott about a homework problem. Students piled into the room, loud and exuberant as the bell sounded. Mr. Abbott's voice was barely audible as he called, "Quickly, let's get settled." He continued to explain the problem to the two boys at his table as a couple of students hurried to finish last night's assignment. One boy called to his group, "I'll go get the books". Students from other tables collected folders and got out books and supplies.

"Okay. I'm going to ask you to get with it," said Mr. Abbott and the noise began to subside. "Turn to pages 57, 58, and 59. What you were doing was going down through number 30 using a calculator to sharpen your skills." He read aloud the answers to the problems as students graded their work and then paused at number 27. "Some of you have mentioned that you weren't sure how to do number 27. They're asking you for a square number." Clearly and carefully Mr. Abbott explained the problem, writing it on the board as he did so. The class was attentive and fairly quiet. Most watched as number 27 was worked out on the board.

After the homework was graded and turned in Mr. Abbott told the class that they would be working on mathematical expressions. He wrote four headings on the board, underlining add, subtract, multiply, and divide. "Okay. Who can give me some other words that mean the same as add," said Mr. Abbott. This was easy and many students raised their hands. Sum, total, increase, more; the list grew long. When students were asked for words meaning multiply they first listed obvious words, like times, but then they became creative. One girl said, "A quick way of addition", and another said, "product". A boy said "fact" and Mr. Abbott wrote factor. "Double," said another boy. "Good," said Mr. Abbott, "and of course we could also have triple and quadruple. We can take any combination of that."

Some students had difficulty in expressing themselves. A boy said "Well, sometimes you can say something like three times three, or three times six. You can say like 3 to a, um, three to a..." As he paused several voices called out. "To a power." The class was becoming more noisy as students began talking among themselves. "We can say to the power of. We'll say n to the power of one," said Mr. Abbott writing this on the board under the 'Multiply' list. "And we'll put a little one up there. We'll say n to the first power and we'll show that's an exponent. Good job," he beamed, "I was wondering if anybody would come up with that." "Square," volunteered a small girl at the back of the room. "We can say square," said Mr. Abbott writing it up. "Square is an exponent. Anyone else? Jordan?" "I don't know how to say this," said Jordan, "but you can say three threes or four fours." "It's the same thing as an exponent," said the teacher pointing to the board.

For the next five minutes students continued to provide words for the lists. "Okay," Mr. Abbott continued, "this is what we're going to do. I'll give you a couple of minutes to quickly copy these expressions from the board onto your paper. Then we'll use them to do the problems on page 63."

Students spent the next few minutes copying the mathematical expressions; some dashed them off quickly and began talking with others at their table, other students wrote carefully and had not finished when Mr. Abbott resumed talking. "Now then," he said, "some of you know that from time to time you need to convert from one unit to another, like, er, time. We use time all the time. We go from hours to minutes and minutes to seconds and we go from hours to days and weeks, don't we? We know how to do that. Well it's the same thing when we're talking about a work day. If we work eight hours a day that is really a rate of work, isn't it? So for one day what would I multiply that by to find out the total number of hours? Krissy?" "Twenty-four," said Krissy. Mr. Abbott moved on and repeated the question, "For one day's work what do I need to multiply, Peter?" "Eight times one," said Peter. "So that gives me eight hours for that first day," said Mr. Abbott. "What if I wanted to work five days?" Mr. Abbott talked to the class about 40 hour weeks and how in some jobs people worked overtime. Students listened with interest as he talked of salary or hourly work and of earning 'time and a half'.

For the next twenty minutes Mr. Abbott described the different problems on page 63 and worked through some of them on the board. He called on several students for answers and some volunteered explanations but other students became restive. They chatted with friends nearby, several drew on their papers and a couple pulled out reading books. Finally Mr. Abbott was satisfied that students were ready to work from the book. "I want you to work quietly on page 63, problems 2 through 30 even, and if you don't finish do it for homework." For the remainder of the period students worked on the assignment. The room was quiet as Mr. Abbott walked around, distributing reward cards and pausing to help individuals. Some students spoke softly to each other but most appeared to be applying themselves diligently to the task and there was little interaction. During the last few minutes several students relaxed, having finished the assignment and the noise level began to rise. As the bell sounded Mr. Abbott called out, "You were great. See you tomorrow."

These examples of the reality of daily life in the three types of classes are in consonance with Berger and Luckmann's (1967) view of the social construction of reality. Basic students placed in the pilot class behaved differently when compared to their peers in the basic classes; also students who scored at around the 92nd percentile in mathematics, and who were placed in the three types of classes, exhibited different behaviors. Thus, the interaction that takes place between students and the mathematical environment in which they are placed may either limit or create opportunities for the construction of mathematical knowledge. This relates to the symbolic interactionist view that humans act towards things on the basis of the meaning those things hold for them.

The remainder of this chapter presents the findings in three sections: effects of labeling classes on the delivery of curriculum and instruction, the importance of social interactions in the learning process, and the opportunities available to students in variously grouped classes for the active construction of mathematical knowledge.

Effects of Labeling Classes on Instructional Practices

Findings in this section refer to the first research question:

> How does the delivery of curriculum and instruction differ in the heterogeneous and homogeneous mathematics classes observed?

The first research question sought to determine whether the delivery of curriculum and instruction differed in the heterogeneous and homogeneous mathematics classes observed in the study, and if so in what ways. Findings to this research question relate to the symbolic interactionist view which states that humans act towards things based on the meaning those things have for them. Of specific interest in this section is the extent to which the grouping arrangements affected the teacher's approach to instruction.

The teacher based his instruction on what he perceived to be the needs of the various groups of students. He was aided in part by a diagnostic placement test that he gave to students in the advanced and pilot classes at the beginning, and end, of the school year.

> Teacher: The placement test tested the basic skills. That would diagnose where they were and what skills they were deficient in and give me some general idea of what was going on in their little minds mathematically.
> Researcher: The basic classes didn't do this test?
> Teacher: The basic did not. We probably should have done it for all groups but we didn't because the basic...well we knew where they were. I had the comps (compensatory students) and everything else.

The mean scores for students in the two advanced classes on the September placement test were 13.03 and 13.27, whereas the mean score of the pilot class was 6.29. When these students were tested again on the same test in May the mean scores for the two advanced classes were 26.9 and 27.6. The mean score in May for the pilot class was 26.09. When students in the pilot class were given opportunities to improve their deficient mathematics skills their scores rose, and were comparable to those of the advanced classes.

The advanced and pilot classes were also given a test, at the beginning and end of the school year, that was prepared by the Florida Mathematics League. In September students in the advanced and pilot classes took the 1991 test and in May they took the 1992 test. This test consisted mostly of problem solving. The mean score for one advanced class was 14.45 in September and 21.37 in May. In the other advanced class the mean was 15.3 in September and 21.43 in May. The teacher believed that scores rose in the advanced classes because students practiced Florida Mathematics League questions in the classroom with him. He believed that those students in the pilot class who had been in the basic fifth-grade curriculum did poorly on this test because they were deficient in their mathematical experiences. They had never been exposed to enrichment classes where students were introduced to geometry, sequencing, and other topics. In the pilot class the mean score in September was 10.67; this rose in May to 15.8.

Neither of these tests were given to the basic classes. The teacher perceived the diagnostic placement test to be unnecessary because he already "knew where they were", and the Florida Mathematics League contest was considered too difficult, even though there were many students placed in basic mathematics classes who scored above the 90th percentile in mathematics. The teacher made decisions about the basic classes as if they were a homogeneous group even though students' mathematics percentiles ranged from the 5th to the 97th percentile.

In many other ways all classes were treated the same by Mr. Abbott. His firm, yet pleasant, demeanor ensured that all students were in learning environments where overt misbehaviors were minimal. Students in all classes appreciated the teacher's friendly personality and the clarity of his direct instruction. However, students in the three types of classes were exposed to very different mathematics curricula. Basic mathematics students used the sixth-grade basal textbook, which consisted mainly of repeating mathematics done in previous years, while students in the advanced and pilot classes had a similar curriculum and used the same advanced mathematics book that was based on algebraic concepts and problem solving. The teacher's approach to instruction was affected by the grouping arrangements, and by the mathematics curriculum in the various classes.

Direct Instruction in the Various Classes

When students were asked in a survey to describe a typical mathematics class their responses were examined by a panel of educational researchers from the local College of Education. The panel commented on the similarity of these anonymous responses. It was difficult to distinguish between the responses of students placed in the various groups:

> We sit down in our groups and take out our math books. Mr. Abbott tells us what we're doing and we get to work. (pilot)
> Mr. Abbott tells us what we are going to do today, then shows us an example, tells us what pages to do and we work. (advanced)
> We come in and get out the supplies and look to the board and sit and listen. (basic)
> We learn a new math skill on the board and Mr. Abbott explains how you do it and then we do the math problems out of the book. (advanced)
> We get point cards and then he explains what we will be doing. Then, when we have homework, he lets us start. (advanced)
> Well, the teacher tells us what to do then we work the problems. (basic)
> He explains how and what to do, goes over it with us and then lets us try. (pilot)

Mr. Abbott's classes were always teacher-directed and he was critical of teachers who limit their explanation time. "I can remember teachers who limit their time for explanation. 'Okay, the first 10 minutes I'm going to go over this and the rest of the period you're going to do that.' I swore to myself I'm never going to get locked into that." Usually all classes began with Mr. Abbott reviewing the work of the previous day; students graded any homework as he read the answers. The next 20 to 30 minutes were generally taken up with direct instruction and during the last 10 to 15 minutes students worked on an assignment. Students from all classes believed Mr. Abbott was a good teacher because he explained things well.

> He breaks it down by steps and explains each step. He explains it til you really understand it. (basic)
> One of the things that I like so much about him is that he always explains it very clearly. (advanced)
> If you really listen to him you can get it the first time he says it 'cos he explains it really good. (advanced)
> He explains it more than any other teacher. (pilot)
> Sometimes he'll explain it one way and then he'll explain it five more ways. (advanced)
> He explains and explains and explains until everyone gets it. (basic)

However, even though this format was regularly followed in all classes there were differences in the teacher's direct instruction.

Differences in Direct Instruction

The first thing that happened in all classes was that Mr. Abbott would go over any homework assignment. If students in basic classes had been given an assignment the previous day, the teacher would go over it the following day with great detail. Students were quiet during this time as they graded the work; those who had not done the assignment now had the opportunity to write in answers as the teacher worked them out. Basic mathematics students seldom raised questions about the homework; they either checked their work or wrote in answers as they were given. The advanced classes usually entered the room boisterously but settled quickly and they also graded any homework with only a few questions. The pilot class were more involved than the advanced classes when homework was being graded; students interacted with each other more frequently and raised more questions about the homework.

Although students in basic mathematics classes needed to improve their skills, they were given less homework than other classes. When questioned about the amount of homework they were given, basic mathematics students stated that they received very little homework. James and Billy both made the same comment, that they liked mathematics because it was easy and they did not get that much homework.

> Researcher: How often do you have homework a week?
> Myneera: Sometimes, not even hardly a week. Five times a month probably.

The teacher believed that this group of students would become motivated if given only a small amount of work to do at home.

> Researcher: Do all the classes have the same amount of homework?
> Teacher: Well, I lessen the amount for the basic class. I'm trying to encourage good habits. If you put it within reach then they may try.

Usually, students in all classes were given the last 10 to 15 minutes of the class period to work on an assignment. Homework was to finish this assignment. The majority of students were able to complete the work during this time. Others chose not to do it.

> Researcher: How often do you actually have to complete the homework at home?
> Megan: (basic) I usually finish in class.
> Jodi: (advanced) A lot of the kids get all their homework done in class and have homework like once a month and other kids drag it out and have homework almost every day.
> Researcher: Why don't they do it in class?
> Jodi: I think they like to talk with their friends.

While appreciating the clarity of Mr. Abbott's mathematical explanations, almost all of the students interviewed from the pilot and advanced classes stated that they regularly studied at home with their parents.

> My mom tries to explain it but she doesn't do a very good job. (advanced)
> Sometimes I don't understand the homework. My mom and dad have been helping me. (pilot)
> I do the homework at home so my mom and dad can help me. (pilot)
> Mr. Abbott shows us two ways and then I go home and my mom shows me two more ways. Then I'll tell Mr. Abbott that my mom showed me these ways and he'll show them to the rest of the class. (advanced)
> See, the stuff we're doing this year, unless I'm really well explained, I have to get help from my parents when I get home. (pilot)

After going over any homework assignment, Mr. Abbott would introduce the topic for the day. During this teacher-directed time students in the basic mathematics class engaged in more off-task behaviors than students in other classes. These were permitted as long as these behaviors did not interrupt the concentration of those listening to Mr. Abbott. Top students in the basic class, who already understood concepts and who quickly finished assignments, were allowed to read or draw. A few students regularly had no assignments to grade when Mr. Abbott was going over the work. They visited the bathroom, or sat with books closed, either occupied with their own thoughts or interacting quietly with a neighbor. However, off-task behaviors also occurred regularly in the advanced classes.

The advanced class also became increasingly off-task during the teacher directed time. Most students listened attentively for the first ten minutes, but then those who understood easily and needed no further explanation, began to talk among themselves or do other things, like writing, reading, or drawing. Students in the advanced and basic classes commented that sometimes explanations continued for too long.

> Sometimes he goes on about things when we check tests and homework. It drags on. I catch on pretty easily and at the beginning of class I listen for 5 or 10 minutes but then at the end of class I get kind of bored with what he's talking about. Pretty much I enjoy it but I get sort of bored and then I just write or do drawings or other stuff. (advanced)
>
> I draw on my paper when he gives instruction because I look at the thing and I say I've done this before and I know what to do. (basic)

In the pilot class there was less uninterrupted teacher-talk than in the other classes. Students became involved in interactions with the teacher and with their peers. There were few interactions in the basic classes where the teacher would clearly and authoritatively state the rules for the day's work and then explain the assignment by working through examples on the board. When working on equivalent fractions, he wrote on the board 1/2 = ?/8 and asked, "How do I do this?" No one responded, although Brian said quietly, "It's 4/8." Mr. Abbott called on Adrian who replied, "You multiply by four." Next, he wrote on the board 12/18 = ?/3 and stated the rule as, "When we go up we multiply and when we go down we divide." When some students were confused about the problem 8/3 = 24/? he said, "If you start with an improper, you end with an improper." The teacher attempted to make the rules simple and memorable. When working on inequalities he reminded students that, "The alligator eats the biggest number."

As few students in the basic classes volunteered answers, the teacher would call on students to answer questions. "What remainder would you get? What's the largest number that will go into top and bottom?" Because the majority of the work was computation, the questions generally required only a few words for an answer. After about 20 to 25 minutes of the class period students would be given a page in the textbook or a worksheet to start in the class and finish for homework. Mr. Abbott circulated during the remainder of the period assisting students and giving reward cards to those who worked quietly.

Similar procedures were repeated in the pilot and advanced classes. However, as the teacher explained the day's assignment in the pilot class, he used different types of questions. Many of the questions were straightforward like those used in the basic classes, but others required students to use higher levels of thinking. "Why would you do it that way?" "Tell me about zero." "Is there another way?"

More students in the pilot class asked questions of the teacher during the direct teacher instruction time than in other classes. They became involved because the work was challenging, for example when the class was working on percentages. The teacher had just worked out that 76% of 350 is 266. Cindy raised her hand and said, "I don't get how you can times 350 by 76 and get 266. If you timesed 350 by 50 it would be more than 266." Again the teacher explained the meaning of the % sign. A couple of minutes later Vanessa interrupted, "But what if it's like 48 is what % of 72? I don't see why you divide." Kirsten wanted to know why they had to know all the different ways. Dean needed to know why his answer of 300 was the wrong one for the question 0.5% of a number is 15. A discussion followed on the meaning of 0.5%.

In the advanced classes interactions during the teacher-directed time were usually between the teacher and individual students. Although the teacher called on all students to answer his questions, only about one-half of the class regularly volunteered responses. The teacher believed that some students were intimidated by the superior knowledge of others in the class and thought they might embarrass themselves.

However, although often reluctant to answer direct questions, advanced students enjoyed interacting in other ways with the teacher; they liked to show him how knowledgeable they were. There existed a special relationship between the teacher and students in the advanced classes. The intern remarked that these students "worshipped" the teacher and Mr. Abbott described these students as "the best of the best. There's a sense of wonder about them". Students in the advanced classes commented that mathematics was fun because the teacher shared experiences with them.

> Jodi: (advanced) He talks a long time, and in my class we have a tendency to wander off the subject.
> Researcher: Is that a good or a bad thing?
> Jodi: Oh, it's fun. Once we were talking and I asked him how can you use this in real life. It was something to do with ratios, I think. And he told us about advertising and got on this big thing all about the advertisements and how funny they were and stuff.
> David: (advanced) He starts out with math and then he talks about things he's done. Like one time he was talking about diving, going down too much, and if you come up too fast you'll get the bends.
> Mary: (advanced) It's fun. It's exciting, not boring. He makes it so you want to do it. Like when he tells us

> stories and stuff. Like when a bird went over a rock and set down one grain of sand every day, that would be infinity.

While the teacher perceived that students in the advanced classes were able to enjoy and relate to these stories, students in basic classes were given few opportunities to hear them. There were other differences in the various classes during the teacher-directed time: The amount of students' mathematical involvement, and the kinds of questions asked by the teacher, differed in the various classes. The way the teacher perceived the various classes influenced the instructional methods used.

Teacher's Perceptions of the Various Mathematics Classes

The way Mr. Abbott perceived his mathematics classes influenced his interpretation of what students needed in the way of instruction, and also what they were capable of accomplishing. He was enthusiastic about the advanced classes, calling them "the best of the best" and pushing them to succeed. Advanced students were given a challenging curriculum and encouraged to work hard on tests and assignments. The teacher told these students that he had high expectations for them. "You're your own motivators, self-starters. These are qualities I want you to maintain. You're here for a reason, and you're supposed to act that way, so discipline should never be a problem while you're in here." These students were treated indulgently. They were encouraged to redo problematic tests and assignments and were invited to phone the teacher at home if they were getting frustrated with the homework. The intern stated that these students "worshipped" Mr. Abbott and were reluctant to accept her help in place of the teachers'.

Mr. Abbott perceived the advanced classes as a homogeneous group. He said that they also saw each other as equals and described them as "going in with a whole lot of advantages...they have no problem keeping up with their work or their homework. They come from good homes that are stable". He said that the advanced classes needed less discussion because they caught on easily to new concepts. He described the students as full of self-confidence, as goal-oriented and motivated, with a wonderful sense of humor. The teacher stated that there were no behavior problems in the advanced classes, unlike the pilot and basic classes, although he said students were sometimes ostentatious and liked to show off.

Although treated as a homogeneous group, the basic mathematics classes were extremely heterogeneous as mathematics achievement scores varied from the 5th to the 97th percentile, as measured by the California Achievement Test. However, this mixed group of students was described by the teacher as "a very basic group, slow, and not willing to cooperate as much as the other classes." The teacher perceived that one way to motivate these students was to continually reassure them that the work was easy and that they would quickly finish it.

At the beginning of the year, the teacher accurately predicted that the basic class would be less fun and less challenging than the other classes, with fewer interactions. He saw it as a class where students would stay sharp on their basic skills. On most days, because the teacher generally perceived the basic class as consisting of slow learners, the work assigned the previous day was repeated over again the next day, with great clarity and detail. This reduced the time available for the introduction of new material. Although individual assistance was given during class time as students worked on assignments, the option to come in for help at other times was not stressed in the basic classes.

When basic mathematics students entered the classroom the teacher also joked and teased the students. However, once the class began there was a brisk use of time and opportunities to interact and misbehave were kept to a minimum. The teacher kept tight control on student behaviors and usually managed to prevent disruptive behaviors from occurring. There were many covert social interactions and off-task behaviors but it was apparent that students knew the level of teacher tolerance. The teacher said that he resented the loss of learning time but did not want to use assertive discipline and have to write the names of misbehaving students on the board. There was a tacit understanding that, in the interest of not wasting the time of the whole class, certain off-task behaviors were permissible.

There was greater absenteeism among the basic students and the teacher commented that you could see the difference in the socioeconomic status of these students; in the way they dressed and in the materials that they had.

> Teacher: There are more personal problems, like their ability to deal with other kids, and their home-life. We found that there were more single-parent families, more things going on at home, less supervision at home, more kids coming in half asleep. You don't find that in your advanced classes, you just don't. The motivation, of course, is always a factor with the basic kid. He or she has always been hard to work with. The willingness is lacking. That's the barrier you're trying to overcome. There's just no incentive for them, intrinsically, to do well in math. There's nothing that motivates them from within.

There was also very little in their mathematical environment to motivate these basic students. They were exposed to repetitive curriculum based on rote learning. Although the teacher recognized the worth of individuals within the basic classes, he found it impossible to meet their singular, intellectual, and social needs within the ability grouped class and he treated them as a homogeneous group. The advanced classes were also perceived as a homogeneous group; as a group of equals coming from enriched mathematical backgrounds, even though many students felt intimidated in the advanced classes. The teacher believed that this group needed less interactive time. "They pick up on new strategies right away". These opposing views affected the teacher's role and influenced the learning environment of the various classrooms.

The teacher found it more difficult to classify the heterogeneous pilot class. He described the class as a mixed bag of personalities, abilities and motivation. He said that some of the attributes of the advanced class existed in the pilot class but not to the same extent. "Some have the self-confidence, some are intimidated." The intern commented that she had difficulty recognizing the low students in the pilot class.

While the heterogeneous pilot class had characteristics of both the other groups, the teacher treated the class more like the advanced classes than the basic classes. In the pilot class the ratio of high-achieving students to low-achieving students was great enough to cause the teacher to teach to the highest level of student ability. Students followed the same curriculum and used the same mathematics book as the advanced classes. The teacher's varying approaches to instruction in the pilot classes accommodated students' differences better than the basic instruction given to students in the basic classes.

In the supportive environment of the pilot class the teacher set high standards, yet made the goals attainable for all, and the mathematical self-confidence of students, who in previous years would have been labeled low or average, increased. In this class there was more teacher-student interaction, as opposed to teacher question, student answer. Because many students in the pilot class had not been exposed to enriched mathematical activities in previous classes they needed to ask more questions. Students in this class regularly engaged in mathematical conversations with the teacher and with their peers.

The Importance of Social and Mathematical Interactions in the Learning Process

Findings in this section refer to the second research question:

How do social and mathematical interactions differ in the classes observed?

The second research question sought to determine how students' social and mathematical interactions were affected when students were placed in heterogeneous and homogeneous classrooms. Of specific interest in this section is the extent to which these interactions affected the learning process, and the extent to which students social and mathematical interactions were constrained by the grouping arrangements.

Although students were placed in the advanced and basic classes on the basis of their mathematics achievement test scores, there were other differences between the groups. The basic classes contained more minority students, most of whom were Afro-Americans. Three of the four black students in the pilot class were high-achieving mathematics students who scored above the 87th percentile on the mathematics section of the California Achievement Test. There were no Afro-American students in the advanced classes; the only minority students in the advanced classes were of Asian origin.

Opportunities for Social Interactions

Socially, manners were different in the various classes. Students in the advanced and pilot classes called out friendly greetings to the teacher and their peers as they entered the room, while most basic mathematics students greeted the teacher only after he had first spoken to them, and their greetings to other students in the basic class often consisted of snatching belongings or a punch. This was usually done in a friendly way, in order to receive recognition. Some students in the basic classes tended to look untidy in appearance, especially when compared with students in the advanced classes, and fewer basic mathematics students regularly used backpacks, lunchboxes, or carried supplies.

Although students in the basic classes knew Mr. Abbott's expectations for behavior, when he was not directing instruction in the basic class, students quickly got off task and began negative interactions, such as pushing each other, name calling, or taking each others' belongings.

> Teacher: The thing that really bugs me is that it's my teaching time and I'm really intense about what I'm

> trying to do. I have a military background, it's no nonsense, it's cut and dried. Let's get the job done but let's have fun doing it. I don't want to go to a kind of assertive discipline situation where you put names on the board.

There were more behavior problems in the basic classes than in other classes as students were tacitly permitted to engage in quiet off-task behaviors and to visit the bathroom as they wished. The same students took the bathroom passes each day; some were often absent from the classroom for nearly five minutes. Mr. Abbott said, "You can see it. It's a constant pattern and you can tell who they are." He reeled off a list of about seven or eight names, all low-achieving students. "Those who are interested won't go right away, they'll wait."

Students who had problems staying on-task commented that they liked Mr. Abbott because, "He'll give you a couple of chances before you get in trouble". James, a low-achieving black student in the basic class, commented that he liked Mr. Abbott, "because he gives you a chance, like if you're talking." This easy-going attitude reduced the number of negative teacher-student interactions and resulted in a less troublesome relationship with low-achieving students. However, the intern had a great many problems when she taught this group and she stated vehemently, "I hate that class. I hate being in it." She described James as having a horrible attitude and said that his table was always fighting.

The intern also found that the advanced classes uncooperative as students preferred to interact with the teacher, rather than with her. The teacher described the advanced classes as more social than other classes. Many students were good friends with others in the class because they had been placed in classes together in previous years. Two students who were new to the school commented, in March, that they had not made close friends with any students in their mathematics classes. Students in the advanced classes tended to come from high socioeconomic backgrounds; there were no Afro-American students in these classes. Advanced students had few opportunities to make friends with those different to themselves.

More than any other class, students in the pilot class said they had made new friends from those in their mathematics class. Because of the mix of students in this class, seating arrangements were changed regularly, and students interacted with most people in the room during the course of the year. Denny, a small white boy, first named Natalie, a black girl, as one of his new friends. These new friendships were fairly tenuous; few students said that they ate lunch with these new friends.

Anna had been in four or five different groups within the pilot class and was presently at a table with two good friends. She appreciated opportunities to be in small groups with her friends but also realized the advantages of having different kinds of students working together.

> Anna: I've had groups where we've had two girls and two boys. That went okay after a while, after we started to understand each other and we were working pretty well. But at first it was like, "I don't want to work with you". But then we started to be more like a group and we learned to work with each other. And it's also good for your social life. If your only around girls that's how you're going to be for the rest of your life but that way you learn to communicate with different kinds of people.

Opportunities for Mathematical Interactions

Students in the basic classes had fewer 'different kinds of people' with whom to communicate both socially and mathematically. The ratio of those scoring over the 80th percentile in mathematics compared to those scoring under the 80th percentile was 3:2 in the pilot class and 1:4 in the basic classes. Few students in the basic classes had been exposed to enriched mathematical activities, either at school or home. Although groups were arranged so that one good mathematics student was at each table as a resource, students had fewer quality mathematical experiences to draw on and share with others.

However, the slower students in the basic classes benefitted to a certain extent from interactive activities. There were off-task behaviors when students worked in small groups but, although more overt than off-task behaviors committed during teacher directed time, they were fewer in number. When asked if the slower students in the basic class learned much from the cooperative activities, Megan, a good mathematics student, said swiftly, "Yeah, I think they did. Some days they didn't do anything but other days they would really do something and try." Myneera commented that James, a low-achiever, was unmotivated. "He can do a lot of problems but I don't think he wants to, that's all, 'cos one time he tried to solve the problem he got real close. He knows how to do it." She said that when she worked on a problem in a group with James and another slow learner, they did learn when they paid attention.

> Researcher: Do they pay attention when you talk to them about it?
> Myneera: About 40% (of the time).

Because of the social norms that prevailed in the basic mathematics classes opportunities to construct mathematical meaning were limited; the reward system encouraged students to work quietly. The top students in the basic classes were the least enthusiastic, of all groups of students, about working in cooperative groups.

The advanced mathematics students were also reluctant to work together. At the beginning of the school year the teacher said, "It's my job to encourage them to work together. They're not going to work by themselves for the rest of their lives, so they might as well learn a lesson in social skills." By the end of the school year these students did work better together when they were given a specially designed cooperative assignment. However, they were still competitive and seldom chose to work with other students; they saw little reason to interact mathematically with their peers.

Once the pilot class had been introduced to the idea of cooperative learning, the average and low-achieving students in the class realized how advantageous this was for them, as the kind of mathematics they were doing was now very challenging. When these students were permitted to interact mathematically their academic self-esteem grew, because they realized that their questions and concerns were being given serious consideration by the teacher and others at their table. The mother of a student in the pilot class who achieved at the 98th percentile, told the teacher how impressed she had been to hear her son explaining a mathematical concept to another student in the pilot class. She found his clear and analytical mathematical communication impressive; he was able to verbalize the clarification of his thought processes. Students of all abilities benefitted from the mathematical interactions that took place in the pilot class.

Students' Need for Various Mathematical Interactions

Although the majority of instruction was teacher directed, students from all classes demonstrated a need for various types of mathematical interactions. The panel of educational researchers from the local Collage of Education found it impossible to distinguish between responses to the request to "Explain the best way for you to learn math." They were struck by the variety of responses.

> When I've been told specifically how to do the problem. (advanced)
> The best way for me is the IBM lab, cooperative learning and video problem solving. (advanced)
> The best way for me to learn is with the math book and a calculator. (basic)
> The best way for me to learn is by remembering how the teacher did it on the board. (basic)
> If the teacher explains it very carefully, then gives the students a try and if they're wrong he explains it. (pilot)
> Having someone show us what to do and then we try it ourselves. (advanced)
> In a group. You learn different ways to do math that you never thought about. (advanced)
> The best way is in a group or on computers. (pilot)
> On computers 'cos it's fun to work on computers. (basic)
> The best way for me to learn math is to be taught as a group and then, if I still don't understand, the teacher help me individually. (pilot)
> When I'm the only one the teacher is talking to. (pilot)
> For the teacher to do a problem or two, ask if there are any questions, then work on your own. (advanced)
> Someone should explain, work one or two problems and then let me try. (basic)

These various responses reinforced findings from the study that a wide range of mathematical interactions were necessary to accommodate students' individual needs. For example, all students from the advanced classes who were interviewed said that they regularly worked on mathematics at home, with their parents and with computers. These advanced students were often exposed to summer enrichment activities and some received individual coaching from tutors. They returned to their classes at school and shared these experiences. Advanced students especially enjoyed the interactions that took place when the teacher shared his personal experiences.

Some of the advanced students were asked if they would have liked to have been in the heterogeneous pilot class. One girl said that she had friends in the pilot class. She said, "All of them really enjoy it. And I think it would be interesting to see, 'cos probably if you weren't as advanced you'd come to a different conclusion. With word problems they might do it a different way and get different answers." Another commented that she came from a school where the students in the enrichment program helped other students. "We would get to help other people and that's what I think the pilot is and that's what I'd like to do." Several of the students interviewed said that they had been in previous programs, in other schools, where they regularly worked with slower students. Without exception, all commented that they found this a rewarding experience.

Yet many advanced students believed that the pilot class would not have challenged them; they perceived the major disadvantage of the pilot class as that of having to go slowly so that other students could catch up. One boy said, "I guess I like being in advanced 'cos if I were in the pilot class they'd have to go back and wait for the other people to catch up and do all the stuff I already know." However, although they were treated as a homogeneous group, students in the advanced classes were also of varying mathematical ability, and many students in the advanced classes regularly occupied themselves by doing other things while slower students received additional explanation.

Differences in Cooperative Learning in the Various Classes

The various classes responded differently to working in small groups. In December, the teacher introduced all classes to the idea of cooperative learning and then, during the time that the intern was teaching, all classes had a unit on cooperative problem solving that lasted three to four weeks. Students were assigned roles and given problems to solve that were specifically designed for cooperative learning. On some occasions the groups were given assignment sheets designed to make them reflective about how well they worked as a group, whether they respected the views of others, and whether all individuals participated. All classes were videotaped as they worked in small cooperative groups.

As the groups in the basic classes began work on the problems, some students pulled back slightly from the table and tipped their chairs back, as if they were disassociating themselves from the problem. The teacher commented that the basic mathematics students were not focused. "They're looking for something to take their minds off math." In the basic class there were many off-task behaviors, especially where two or three low-achievers were at the same table. Other groups in the basic classes became involved with the problem, but the only groups that found solutions had help from the teacher.

In the advanced classes when students were placed in cooperative learning groups some students' behavior was inhibited. They were slow in initiating discussion and appeared embarrassed; they sat and waited for others to speak and tended to sit upright in rather formal postures. There was less noisy interaction and few off-tasks behaviors in the advanced classes. Students were quieter and more thoughtful. Frequently two students in the group would discuss the problem together while the others worked on it independently. Individuals shared how they found the solution to problems but when the whole group was unable to move forward there was less interactive discussion of possible strategies than in the pilot class.

When engaged in cooperative learning, students in the pilot classes were noisier, often leaning across the table and kneeling on their seats. Groups in the pilot became involved, even though occasionally two or three of the boys were off task for part of the time, as they leaned back in their chairs and called out to students at other tables. However, these behaviors were of relatively short duration because most other students were on-task, and within a few minutes students were concentrating again on the problem. When the groups found it difficult to produce a solution, off-task behaviors increased among the slower students.

Groups of students in the pilot and the basic classes worked on the same cooperative learning exercise. The intern had discussed with the classes several strategies that they might chose to use. These included 'make a table', 'guess and check' and 'look for a pattern'. One of the problems was: In the jungle of Ip there are red, gold and silver Spiffle trees. There are eight times as many gold trees as red. There are six fewer silver trees than there are red. There is a total of 1234 Spiffle trees. How many are there of each? Each student was placed in a group of three or four and was given a card that contained part of the problem; each student was also assigned a role, recorder, calculator or summarizer.

In the basic class, Leah, Sally, and Mick, worked together on the problem. One member of their group was absent. Leah scored at the 90th percentile in mathematics, Sally was an average student and Mick had very poor mathematics skills. The boy who was absent had a mathematics score at the 20th percentile. This group worked hard at the problem but found it frustrating. Leah and Sally read their cards aloud as Mick laboriously wrote down the information. Sally directed Mick's writing.

> Sally: O.K. That's fine you don't need to write any more, just the question.
> Mick: What strategy should we use? I think we should look for a pattern.
> Sally: Yes. I think we should make a table and look for a pattern.
> Leah: Read off the clues.
> Sally: Write them down.

Mick haltingly read the clues, calling them 'Spiffley' trees. Leah said 'Spiffle' softly and Mick repeated this as he wrote it down. Although they started well, with enthusiasm, the group soon ran into difficulties.

> Leah: (frowning) There are six less silver than there are red.
> Sally: I don't know.
> Mick: We know what the final answer ends up being. I think a pattern.
> Sally: Write it down on scrap paper.

Mick headed the scrap paper 'Guess and Check' and began writing the clues over again. Sally told him not to write the clues again but she and Leah watched resignedly as he did so. When the intern came near Sally said, "Ms. Dee, we don't know how to do this." Ms. Dee read the clues to them again and said, "start by guessing the number of the red trees. They're the ones you know most about." For the next ten minutes the three students played around with very different numbers. They tried numbers ranging from 1062 to 134 for the red trees. Mick lost interest and began making rude comments to Myneera who was seated at a neighboring table. "You're like a monkey. You've got a hairy face." At one time Leah and Sally thought they had the solution and became excited, but when Ms. Dee checked their solution it was wrong. As she walked away she told them to guess the red trees first at about a hundred. "Shall we try one hundred?" Sally asked Leah as she reached for the calculator. As Ms. Dee called for attention and began talking through the problem with the class, Leah and Sally continued working quietly. "We've got it!" said Leah triumphantly, and she showed Mick how to record the solution on the paper.

Top students in the basic class commented that these problems were the greatest challenge they had all year. The teacher had placed one of the top students at each of the tables. More than any other group, in any of the classes, these top students disliked cooperative learning, stating that they preferred to work on their own. Once the unit was completed there were few opportunities in the basic classes for cooperative activities, although students did share solutions to problems. Myneera was one of the top students in the basic class.

> Researcher: Give me examples of other times that you worked as a group?
> Myneera: When we share you mean?
> Researcher: Yes, that's exactly what I mean. Tell me what happens then.
> Myneera: It's like when me and Yolanda finish first and Jamie and Sam would correct it off our papers 'cos usually our papers were right.

In the pilot class Jason, Tom, Anna and Beth worked together on the problem about Spiffle trees. This group consisted of two good mathematics students, and one average and one below average student. Tom had the calculator, and Anna read problem while Jason wrote down the clues. Tom decided to try 120 for the number of red trees. Anna read aloud that there were six fewer silver than red. Tom began to put 120 minus 6 into the calculator, but Beth said, "You don't have to do that. It's 114." "There were eight times as many gold trees as there were red," said Anna. Jason was busily writing these guesses. "That's not enough," said Beth. "Try 127,"suggested Anna, but when Tom multiplied this by eight the number was too high. The group realized that they were close to the solution and became excited, leaning across and watching closely what was being written down and worked on the calculator.

> Beth: It's got to be 124.
> Anna: No, it could be 122.
> Beth: No, it has to be 124. Look, when we tried 127 it was 1254, and when we tried 125 it was 1244. We want 1234.
> Anna: O.K. What's the answer.
> Tom: (busy on the calculator) 124
> Anna: No, you have to say how many of each.
> Jason: I'll write these down.
> Beth: Now let's just add to be sure.

This class was told to write their own problem when they finished. The boys now sat back and watched as Beth and Anna wrote this problem: There are 524 mice in an orphanage.· They only have 6000 pieces of cheese. How many hours will it take them to eat all the cheese if they each eat one and one-half pieces each day? Beth and Anna worked out the solution as Tom and Jason began flicking pieces of pencils at each other. Although the noise level in the room was now at the point where the teacher threatened to keep the class for part of their lunch hour, students had been working well together until the last few minutes of class.

One girl from the pilot class, who had been in an advanced class the previous year, said that she can get the work done faster on her own but she likes working with a group because "sometimes it asks for more than one way to solve a problem, and if I only know one way someone else might know another and if you need help they help you. I like that." Another girl said that an advantage of the pilot class was that there was less competition than in the advanced classes, "where everybody's as smart as everybody else, so you always want to try and get the highest grade in class". It appeared that there was more need for interactive discussion in the pilot class where the majority of students had not been exposed to an enriched curriculum.

Early in the year the teacher described students in the advanced classes as very competitive.

> Teacher: They are very goal oriented. The disadvantage is that they're reluctant to ask questions. They might embarrass themselves because they might appear somewhat dumb to the rest of the class. Some that are not as competitive as the others feel intimidated. They don't always want to compete. The other thing is that those who work well independently are reluctant to work in cooperative groups. They say, "I don't want to work with him."

In February all students were given a survey in which they were asked whether they preferred to work on their own or with a group. Responses from the pilot and advanced classes were very similar. In the pilot class 68% of students preferred to work in a group, and in the advanced class 65% of students preferred this method of working. Basic mathematics students were more evenly divided; almost half preferred to work on their own. Of the one-third lowest achievers in the basic class 80% preferred to work in a group. However, of the top one-third of students in the basic mathematics class 80% stated that they preferred to work alone.

Although after working through the unit on cooperative learning advanced students began to enjoy working in small groups, they were not as eager to seize opportunities to work together as students in the pilot class. Also the kinds of interaction that took place within these groups was different from those of the pilot class. Students liked to do the problem individually and then share their solutions rather than have an initial sharing of ideas. Advanced students worked competitively within the group; their primary role was to find the solution before others in the group.

> Researcher: Do you like working on your own or do you like working with a group?
> Kim: Um, well, usually I'd rather do it individually 'cos I know what I'm doing and I don't have to conference with other people and make sure.
>
> Jodi: If I do it by myself I just like to get it done but when I'm working with a group that usually makes me think more and I come out with more answers and I usually feel more satisfied at the end.
> Researcher: Did you enjoy doing the cooperative learning unit when Ms. Dee (intern) was here.
> Jodi: Oh yes, very much.
> Researcher: When else have you done cooperative learning this year?
> Jodi: Not very much. That was about the only time.
> Researcher: What about when you watched the videos?
> Jodi: Oh yes, and also when we're doing word problems, Mr. Abbott says, "If you're quiet you can work together."

In all classes Mr. Abbott regularly told students that they could work together if they were quiet. Because classrooms interconnected and teachers were concerned about noise levels, Mr. Abbott constantly reminded his classes to work quietly; classes responded differently to this admonishment. Students in the pilot class were most mathematically interactive; they found it difficult to work quietly as they needed to ask each other for advice and suggestions. In the basic classes, where students were mainly working on computation, and also in the advanced classes, the majority of mathematical interactions consisted of asking others for checks to solutions. Although noisy during the teacher directed time, once they started work students in the advanced classes were the quietest of the groups because they worked independently.

> Researcher: When you're not doing special cooperative learning problems, how do you help each other when you're sitting at your tables?
> Jodi: (advanced) Well, sometimes it's really something quite small, like, "Are we supposed to do the evens or the odds?" And there are little things where it would be a big waste to go all the way up to Mr. Abbott. If you see they've already done the problem you can ask them, "I don't understand the directions. What are we supposed to do?"

The findings showed that different patterns of social and mathematical interaction developed in the various classes as the year progressed. There were fewer positive mathematical interactions in the basic classes; interactions between students were often negative and the teacher was thrust into an authoritative role. In the advanced classes students enjoyed interacting mathematically with the teacher rather than with their peers. In the pilot class social interactions combined with mathematical interactions to contribute to the learning process. The many questions and discussions that regularly took place in the pilot class, between students, and between the teacher and students, were essential because of the kind of mathematics in which students were engaged.

Opportunities Available to Students for the Active Construction of Mathematical Knowledge

Findings in this section refer to the third research question:

> What constitutes meaningful mathematics in the various classes?

The categories of findings that converge under this major heading are all centered around the third research question that addressed such issues as a consideration of the nature of mathematics and want it means to engage in meaningful·mathematical activity. This was very different in the various classes; some students worked regularly on basic computation, while others interacted with videos and computers and were involved with algebraic concepts and problem solving.

In all classes the routine of daily life in the advanced, pilot, and basic mathematics classes was similar in many ways; all students were warmly greeted by Mr. Abbott and were assigned to a seat at a table with three other students. Classes were also similar in that they consisted mainly of direct instruction and students perceived this teaching strategy as a major strength of Mr. Abbott. Students, of all abilities and from all classes, described Mr. Abbott as a very good mathematics teacher. When asked specifically to define what constituted a 'good' teacher, students commented that Mr. Abbott explained things well, that "he explains it until everyone knows it." Although similar in these ways, findings from observations conducted during the entire school year found the content of these mathematics classes to be very different.

Differences in Sixth-grade Mathematics

From the following descriptions of the kind of mathematical activities in which the three types of classes were engaged it is possible to determine how opportunities for the active construction of mathematical knowledge were created, or limited, by the grouping arrangements.

Mathematics in the Basic Classes

The teacher was unenthusiastic about mathematics in the basic classes. At the beginning of the year he predicted that it would be less fun, less challenging, and less interactive than the other classes. At the end of the school year, students in the basic mathematics classes were given a final examination that was compiled by the textbook publisher.

> Teacher: Here's the Heath test that was my final. I said, "Do what you can" and there's some I omitted 'cos I thought they didn't have enough time or I didn't think we'd spent enough time on it.

The sixth-grade edition of the Heath mathematics book was the textbook for the basic mathematics classes. The book was very similar to the fifth-grade edition with only a small amount of new material. In March and April, basic mathematics students were questioned about what they had learned in mathematics during sixth-grade.

> Researcher: What was the last new thing that you learned?
> Myneera: The last thing we did was fractions.
> Researcher: You hadn't done fractions before?
> Myneera: Oh, yes. We'd done them way back in elementary school.
> Researcher: So it wasn't really new. Is there anything you've done this year in math that you've never done before?
> Billy: No. Not yet.
> Myneera: (long pause) Um, No. I can't think of anything.
> Megan: We do practically the same things but they're a little bit harder.

Students from the basic mathematics classes were unable to think of anything new that they had done in mathematics during the year, except for two students who mentioned cooperative learning. At the beginning of the year the teacher was aware that many students would not be challenged in the basic class. He said that several of the top students came to him and said they were bored. The teacher recognized the limits of the basic curriculum.

> Teacher: I said, "It won't be fun and I'm not telling you that it's going to be fun and it won't be as challenging as the other class. There won't be the interaction you have in the other class but if you bear with me we'll try to see that (you) make some gains and (you)'ll keep sharp on the basic skills." And basically they did. I gave Myneera some of my advanced work and they did well in the computer lab because I would let

> them work at their own pace. And that's about all you can do.

The teacher supplemented the textbook with worksheets, most of which were supplied with the teacher's edition of the textbook. Several of the basic mathematics students commented that they liked the worksheets.

> Researcher: Do you like doing worksheets?
> Brian: Oh, yes. It's better than the book because you don't have to write down the problem. You can just fill it in.

Billy, a small black boy who was an average student in the basic class, commented that the book was easy and the worksheets even easier.

> Researcher: Do you think you should be doing harder work?
> Billy: Yeah. I don't really want it to be but I need it.

Although worksheets were regularly used in the basic mathematics classroom, they were primarily used for homework. Worksheets were seldom used in the pilot or advanced classes.

> Researcher: You use more worksheets with the basic group.
> Teacher: Oh, yes, definitely. They expect homework. The advanced and pilot would go back and look at examples, but the basic wouldn't do that.

Some weeks worksheets were not used at all, for example when students were working on a cooperative learning unit and no homework was given, but most weeks at least two worksheets were assigned.

> Researcher: What kind of work do you do mostly in this class?

> Myneera: Actually, it's not really out of the book. We do a lot of basic worksheets. They're not hard at all, just practice.
> Researcher: Do you do these worksheets more than you use the book?
> Myneera: It's hard to say. We do it after and sometimes before. We do two. Probably about the same.

Students in the basic classes received few opportunities to actively construct mathematical knowledge. They were unmotivated by mathematics that they were told was easy, and which that had already done in previous years.

Mathematics in the Pilot Class

All students in the pilot class used the same mathematics book as the advanced classes. This consisted of challenging work and involved higher-order thinking, unlike anything the low-achievers had been exposed to in previous years. These students were motivated by this challenge; they raised questions and demanded explanations, both from the teacher and from other students. The teacher commented that students were often more willing to listen to a peer than to him.

> Teacher: So I think there are advantages for the basic kids because they're almost forced to interact, yet they're interacting with someone they'll listen to. The thing about the basic kids is to keep them tuned in. They've never had success in math and their attention span is short. That's why when they're doing cooperative learning and when they have the calculator or when they're being the recorder, you know they're getting more because if they were doing it on their own they'd be out in left field, or in the north 40 somewhere, and you'd have no idea what they're thinking.

While all students in the basic mathematics class, regardless of ability, commented that the work was easy, the low-achieving students in the pilot class found the work challenging. Denny, a low-achieving student in the pilot class, thought some of the work was the same as the previous year but listed many topics that were new to him, like ratios and percentages. He stated that the work had been hard but fun, and sometimes boring. Mr. Abbott stated that some low-achieving students in the pilot class, who had only been exposed to the basic fifth-grade curriculum, were often intimidated by the mathematics they were expected to do in the pilot class.

> Teacher: I had kids in the pilot class who wanted out. They said, "It's too much work. I'm struggling with it."
> Researcher: Was the work really that hard?
> Teacher: See, to them the ability to think critically and analytically wasn't there. They'd never had to do that. Just hand them the basic worksheet, sit down, do the skills, punch out the numbers and that was it. So now they're having to think about divisibility rules, order of operations, and commutative and associative properties, and all of a sudden you start throwing these at them in different patterns and then there are so many problems to solve and they have to think, "What's the strategy?" They don't like having to think like that. "Just give me the problem and I'll work it and I'll be happy. I'll know I can get it either right or wrong."

The teacher was asked which students benefitted most from being in the pilot class.

> Teacher: The average kids came up. The low kids made gains. They were exposed to higher level math. They became associated with skills they would not have gotten in the basic class. The advanced kids learned to do things that they wouldn't have learned in the advanced class and that's working together or assisting others.

Some of the top students in the pilot class blossomed in the role of group leader. They gave advice on how to work mathematics problems in the organized setting of cooperative groups, and also during regular classroom assignments. Some of these top students sometimes asked to take their groups, or individuals, into the small neighboring room to work on problems together. Catlin thrived on this additional responsibility. She had been in an advanced class the previous year and was now in the pilot class. She thought the mathematics class was fun because they did not always stick to the mathematics textbook. She said that when the class was reviewing she found the work easy but overall she described the class as "fun and challenging. It's been a lot harder than it was last year."

Anna, an average student in the pilot class, commented enthusiastically on some of the work the pilot class had done.

> We've worked with the problem solving videos. Those were great, really exciting and fun 'cos we tried to see who could work it out the quickest and we've done little card problem solvings and we've worked on ratios and different kinds of things I never saw before in my life. I knew you could do that sort of math but I've never see it til this year. I didn't expect to do it til 7th or 8th grade.

Students in the pilot class had more opportunities to see a broader picture of what mathematics could be, than students in basic classes. Students of all abilities benefitted from being in this heterogeneous group, and the mathematical interactions that resulted from the mix of students' abilities contributed to the learning process of all students.

Mathematics in the Advanced Classes

Although all students in the advanced classes scored above the 92nd percentile on the mathematics section of the fifth-grade California Achievement Test, there were still considerable differences in their mathematical ability. Some students caught on to new concepts very quickly; they said that after the first five or ten minutes of class they did not need to pay close attention. They would "write or do drawings or other stuff". One student said, "We're sort of cocky about what we know. It's sort of like we're cocky and we think we're above everybody else."

The advanced classes were often quite noisy during the teacher directed time. Some of the noise was caused by students asking questions; they raised their hands but were also often permitted to call out. However, some of the noise came from students who lost interest when the teacher answered individual questions.

> Kim: I sort of listen the first five minutes and then later, afterwards, I'm getting sort of bored 'cos he does sort of individual stuff. And the rest of the class, it's like normal voices because we're all talking to each other and writing and all sorts of things are going on until he finally gives the assignment and we all sort of quiet down and get to work.

One advanced student, who believed that she could have been more challenged, planned to jump a grade level and move into algebra the next year. She said that she was pretty fast at doing the homework assignment but there were others "who are really fast and they usually get it done but I have about a couple of problems to do." Other students said that they liked to take the assignment home; some did this because they needed help from parents, some wanted to work on mathematics at home. Others preferred to talk with friends during the last 10 minutes of class.

Students in the advanced classes said that they had learned a lot of new mathematics during the course of the year. Of the new material presented to them, they most enjoyed the interactive videos and the computer problem solving.

> Mary: We've learned so much this year. I don't even know how well I'm going to do on the finals 'cos I can't remember stuff from the beginning of the year. We've done so much.
> Researcher: About what percent of the work would you estimate was new this year?
> Mary: I'd say about 75%.

Like the pilot class, the advanced classes were introduced to a broad picture of the nature of mathematics. They used the same mathematics book and enjoyed interacting with videos and computers. They differed from the pilot class in the number and quality of mathematical interactions that took place during mathematics classes. Advanced students were competitive, and while they enjoyed mathematical exchanges with the teacher, they saw little reason to have mathematical interactions with their peers.

Use of Computers, Calculators and Videos

During the course of the year all classes had a unit on using the Texas Instrument Explorer calculator, and all classes regularly used calculators when they were working in cooperative groups. The calculator increased self-confidence and helped the slower students to be more comfortable when working on problem solving or working in a group. Students in basic classes regularly used calculators to help them with computation and also to correct answers to the previous day's assignment.

> Teacher: If you take the calculator out of their hands and give them a worksheet you can consistently look at your low stanine kids and they've stopped dead in the water. They don't budge, in fact they're looking to see, "What did you get for that?" so that they can write down the answer. But if you give them a calculator they'll try.

The use of the calculator in the basic classes was of mixed value. It was of great benefit when students were working on cooperative learning or problem solving. However, the main purpose of the basic curriculum was perceived to be the improvement of students' basic mathematics skills. The regular use of calculators for worksheet and textbook assignments that were mainly computation, and for correcting arithmetic problems, just resulted in students doing 'busy work'.

Between the beginning of the school year and Thanksgiving the pilot class worked on basic skills in the computer room approximately 18 times. The advanced classes each worked there 6 times and basic classes each worked there 3 times. During the remainder of the year, the advanced classes had 15 lessons with computers, and the pilot and basic classes each had 12 lessons.

The favorite mathematics activity of the advanced and pilot classes was the mystery videos. Students watched the mystery on the television monitor and then used cooperative activities to solve mathematical problems. The basic mathematics classes did not have these video activities.

> Teacher: The video is a great new technique but there's a lot of skills that the basic kids won't know...Myneera and some of the kids could have done it, but the majority of the tables couldn't have done it. Some had no idea how to read maps, or map scales, and they'd think the Fibonacci sequence is some sort of spaghetti.

Although many students in the basic classes were capable of doing the video and computer problem solving programs, the grouping arrangements denied them access to these activities.

Students' Perceptions of their Mathematics Classes

During February, students in the advanced, pilot, and basic mathematics classes were given an attitude survey that consisted of three parts (see appendix). Responses to questions were tallied and compared across all grouping arrangements. Also, responses were analyzed for high, average, and low groups within each class. Students were asked to respond to nine questions about their mathematics classes with a few sentences and then circle three words that described how they usually felt about mathematics classes. The word 'easy' was used most often, by 64% of basic mathematics students, to describe their basic mathematics class. In the pilot class 'challenging' was circled most often, by 62% of the students, to describe their heterogeneously grouped class. Advanced students used 'fun' most often to describe their mathematics class. Of the students in the advanced class 69% circled the word 'fun' and 66% circled the word 'interesting'.

In response to the questions that required written answers, the majority of students, in all classes, had positive attitudes toward mathematics. However, when asked which class they looked forward to most each day 16% of the basic mathematics students, 39% of the pilot class, and 28% of the advanced students chose mathematics. All students reported that direct instruction was the teaching strategy most frequently used in their mathematics classrooms. Sixty-seven percent of the basic mathematics students said that they learned best from this method of instruction. However, approximately half of the pilot and advanced classes stated that they learned best from using other methods, such as technology and cooperative learning.

Students were also asked to react to 40 statements about mathematics by circling a letter A through E to indicate whether they agreed or disagreed with the statement, or had no opinion. These statements were loosely based on Riedesel's Inventory of Children's Attitudes Toward Mathematics. Questions were concerned with factors that have an effect on students' outlook concerning mathematics.

Twenty-six questions were concerned with specific attitudes toward mathematics: liking for mathematics, usefulness, anxiety level, and perceived ability. Fourteen questions were concerned with other variables: the teacher, peer groups, and methods and materials.

From this inventory of students' attitudes toward mathematics it appeared that the basic mathematics students had a favorable attitude toward mathematics, primarily because they liked the teacher. All questions associated with the teacher were very positive. However, students disliked the textbook, found most of the work repetitive and too easy, and thought that the little homework they had was boring. Mathematics was not a favorite subject of most of the friends of students in this group. Observations supported findings from the survey; the brightest students in the basic classes displayed little enthusiasm for the mathematics they were doing. For example, although Adrian had scored at the 97th percentile in mathematics, he was quiet and passive in the basic mathematics classroom. He only volunteered answers when asked and spent much of the time in off-task behaviors, such as day-dreaming and drawing. Other top students in the basic class usually spent part of the period drawing, reading, or writing. Megan usually sat at the table next to the hallway with her feet on the table and said little during the class period. Several of these students said that they should have been in the advanced classes; they perceived the basic mathematics classes to be inferior to the advanced and pilot classes. They were unchallenged and believed they were misplaced.

Attitudes toward mathematics were very positive in the heterogeneous pilot class, especially those of the students that could be called average, the middle one-third of the group. Although only half of this average group said that mathematics was easy for them, they unanimously declared mathematics to be fun and not boring. Only two students in the pilot class stated that they did not like mathematics; 83% liked mathematics best when instructional strategies other than the textbook were used. Only three students did not like working in small groups; 81% of the pilot class wanted more small group work during mathematics classes. In all classes the great majority of students stated that they liked working with computers.

All but one of the advanced students agreed that the teacher made mathematics interesting. When compared with other classes, more students in this group preferred to work a long interesting problem than a short easy one. Fewer of the advanced students disliked the textbook, although 84% of the students preferred mathematics when the textbook was not used. About 75% of the advanced students said that wanted to work more frequently in small groups.

The panel of educational researchers from the local College of Education were asked to react to students' responses to the question, "What is the most favorite thing that you like doing in your math class?" Members of the panel were asked to give their opinion, positive or negative, concerning whether students' choices constituted meaningful mathematical activities. The panel was also asked to assess whether the student making the response to this question came from the advanced and pilot classes, or from the basic classes.

Overwhelmingly, the panel reacted negatively to responses that stated students liked worksheets and basic computation. In the panel's opinion meaningful mathematical activities were those that demonstrated the use of higher levels of mathematical thinking, like problem solving, or interactive activities like cooperative learning or interacting with technology. The panel was accurate 81% of the time in attributing students' responses to the advanced and pilot classes, or the basic classes. The panel's assessment that many students in basic mathematics classes enjoyed low-level kinds of mathematics supported the findings of the researcher.

Findings from the study indicated that students in the basic classes were not as often exposed to a variety of teaching and learning strategies, when compared with the other classes. Therefore they had fewer activities from which to select a favorite. Basic students had become conditioned to low-level work; they had become comfortable with the non-challenging work that determined their grades, even though they knew that it was too easy. One student remarked "I don't really want it to be (more difficult) but I need it." Another student said, "I feel like it's so easy. I can do this. If I come upon a hard thing I won't think that but now, once I'm done, I'm going, 'I'm done, this is it, that's that'. Fractions, multiplication, adding, that's so simple! Something you can learn in another grade like 5th or 4th." The different kinds of mathematics to which students were exposed affected their motivation and their mathematical self-esteem, as well as influencing the kinds of mathematical interactions that took place in the classroom.

Summary

Opportunities to actively construct mathematical knowledge differed in the various classes even though it was apparent that the teacher had a constructivist perspective. He stated that he wanted to move away from arithmetic and rote learning, and use many different methods of teaching, including the latest technological ideas; his desire was for an interactive curriculum based on problem solving and cooperative learning. However, the teacher found it difficult to achieve these goals when students were ability grouped for instruction, because of the assumption that these classes were homogeneous; within the ability grouped classes all students were treated the same.

Students in the basic mathematics classes mostly worked on computation. They were exposed to a limited curriculum that hardly changed from the previous year. Students of all abilities in the basic class described the mathematics as easy and found it difficult to think of anything new that they had done in mathematics during the course of the year. The top students in the basic classes who already knew how to do the work often used the teacher-directed time to draw or read. These students believed that they should have been placed in an advanced class. Not even the 20 students in basic classes who had scores above the 90th percentile in mathematics participated in the Florida Mathematics League Contest, or had the opportunity to take part in solving the mystery videos. These students were unchallenged in basic classes and lacked the stimulation of others with good mathematical skills.

Constructivists believe that all students should understand the broad nature of mathematics. In the advanced and pilot classes students more frequently received this 'big picture'. Through the teacher's stories and anecdotes, students saw mathematics as a useful way of describing the real world. Students in these classes received a curriculum that gave them a broader idea of what sort of things make up the subject of mathematics. Constructivists believe that mathematics should be more natural and should fit better into the context of students' lives. This occurred less frequently in the basic classes where the curriculum consisted mainly of rote learning and where students were encouraged to always follow certain rules.

Another goal of constructivists is to get each student to believe that mathematics is worthwhile and rewarding and to see mathematics as a reasonable challenge. Students in the advanced classes enjoyed their classes. However, more than twice as many of these students strongly agreed that it was the teacher who made mathematics interesting compared to the number of students that stated that mathematics itself was very interesting. However, all students in the pilot class described the mathematics they were doing as challenging. Denny, a low-achiever, said that he enjoyed the class but thought that his grade might have been better had he been in the basic class as it would have been easier, with easier tests; Laura, another low-achiever who was making D's and F's in other classes, was making A's and B's in mathematics. Even the top students in the pilot class, who had done some of the mathematics before in their advanced fifth-grade class, said that the pilot class was challenging; they said it was harder than the previous year when they had been in advanced classes. Those with friends in the basic classes stated that they were envied for the mathematics that the pilot class was doing. One girl in the pilot class quoted her friend in the basic class, "I wish I was in the class you're in 'cos compared to your stuff ours is completely boring."

The advanced and pilot classes had more opportunities than the basic classes to construct their own mathematical knowledge. They were asked to think creatively about what they were doing. Is this the only way? What other way can we do it? Is there another solution? However, in the advanced classes there was an assumption that students grasped new concepts easily. The intern commented, "You explain it once and they know it and you can move on."

> Teacher: In the advanced classes they pick up on it relatively quickly, so you don't have to involve the class or the tables in as much discussion. So we don't have to waste as much time in trying to see what's going on, what's the strategy or the approach, what's the question that they're asking for.

When classes were labeled 'Advanced' or 'Basic' they were perceived and treated as homogeneous groups by the teacher and this often had a deleterious effect on the kinds of instruction students received and the kind of mathematics to which they were exposed. While both the basic and the advanced classes tended to be more traditional and teacher directed than the heterogeneous pilot class, the pilot and advanced classes were exposed to a greater variety of instructional strategies than were the basic classes. This resulted in unequal opportunities for students to actively construct mathematical knowledge. Constructivist methods of teaching fitted most naturally into the heterogeneous pilot class and were most difficult to apply in the basic classes.

CHAPTER 5
CONCLUSIONS AND IMPLICATIONS

The purpose of this study was to examine, in terms of constructivist theory, the opportunities to construct mathematical knowledge available to sixth-grade students when a teacher's mathematics classes were variously grouped for instruction. Constructivist theorists hold that students actively construct mathematical knowledge when they are placed in mathematical environments that encourage complex, interactive, mathematical activities. This study responded to a documented need for descriptive studies that look at how various grouping arrangements shape learning opportunities (Dawson, 1987; Gamoran, 1987; Gamoran & Berends, 1987; Oakes, 1990; Slavin, 1987). Slavin (1987) stated that without descriptive studies of this type the effects of various grouping practices are only speculation.

Research on ability grouping has measured its effects on student achievement and has examined also the affective results of placing students in homogeneous and heterogeneous groups. Research that describes how instructional practices that take place in homogeneous and heterogeneous classes are interpreted by students and by their teacher has been virtually nonexistent. This study was undertaken in order to contribute knowledge to educational leaders, practitioners, and researchers, on the effects of various grouping practices on students' ability to construct mathematical knowledge.

The three major findings that emerged from analysis of the data were as follow:

1. The interaction that takes place between students and the mathematical environment in which they are placed may be responsible for differences in students' mathematical achievement.
2. When the variously grouped classes were compared, it was apparent that labeling classes affected the teacher's approach to instruction.
3. Social and mathematical interactions in the various classes differed. Interactions affected the learning process.

Berger and Luckmann's (1967) symbolic interactionist views on how reality as a social construction are highly applicable to this study. This study has as a major emphasis an investigation into the opportunities to actively construct mathematical knowledge available to sixth-grade students in variously grouped classes. Data collected through observation, in-depth interviewing, and videotaping provided strong links to the constructivist questions that guided the research. When the data was examined in terms of constructivist theory, the following three broad categories emerged: constructivism in sixth-grade mathematics classrooms, symbolic interactionism and the delivery of instruction, and the social construction of reality through interactions.

Constructivism in Sixth-grade Mathematics Classrooms

Findings support the constructivist ideas on the nature of mathematics that provided the underlying rationale for reports such the NCTM's Standards (National Council of Teachers of Mathematics, 1989) and Everybody Counts (National Research Council, 1989). In the current study the teacher and his students were observed as active constructors of their mathematical knowledge. Notice was taken of the constraints that limited students' mathematical constructions. In the variously grouped classes, the nature of mathematics, or what it means to think mathematically and to engage in mathematical activity, was perceived differently by the teacher and students.

A major finding of this study is that the interactions that take place between students and the environment in which they are placed may be responsible for differences in students' mathematical achievement. Six specific findings support this conclusion:

1. Students in the pilot and advanced classes were exposed to a broader mathematics curriculum than students in the basic classes.

The curriculum of the advanced and pilot classes was based on algebraic concepts and problem solving. Students in these classes also used innovative computer and video programs. Doing these kinds of mathematics involved using complex processes, and students were required to think critically and analytically. Students in the basic classes worked mostly on computation. To these students mathematics meant following certain rules.

2. Students in the basic mathematics classes were introduced to only a small amount of new material and faced few academic challenges.

The curriculum of the basic classes repeated that of previous years; students worked mainly on arithmetic. Students were unable to name any new topics that they had learned during the course of the year. These students were unmotivated because they were offered basically the same curriculum as that of previous years. Mathematics in the basic classes consisted mainly of easy, low-level, repetitive worksheets and textbook assignments. Basic mathematics students were given less homework than students in other classes.

3. As a result of the constructivist activities that occurred in the pilot class, the mean, end-of-the-year achievement test scores of the pilot and advanced classes were comparable.

All students in the pilot class, regardless of mathematical ability, were exposed to the same challenging curriculum. To make sense of this experience they drew on the resources of both the teacher and their peers. In this supportive environment students were given new mathematical tools and were encouraged to find new ways to solve problems. The teacher created a mathematical environment where students were encouraged to construct their own mathematical knowledge individually and collectively. After taking the diagnostic achievement test in the fall, students in the pilot class worked in the computer room once or twice a week for twelve weeks to improve their skills. In May, when students were retested, the mean score of the pilot class increased by 19.8 points. This score was similar to end-of-the-year scores of the advanced classes.

4. When students were organized for mathematics on the basis of their perceived abilities students were sometimes misplaced.

In the advanced classes the lowest mathematics score from the fifth-grade California Achievement Test was at the 93rd percentile. Some students in the basic classes had higher mathematics percentiles. Twenty students in basic classes had scores at the 90th percentile or above. These students regarded their classes as inferior to other mathematics classes and believed that they should have been in the advanced or pilot classes.

5. A greater concentration on the textbook, in both the advanced and the basic classes, promoted formal thinking about mathematics.

Students in these classes worked independently more often than students in the pilot class, and were less eager to take part in group problem solving. There was more rote learning in the basic classes where students were encouraged to memorize and follow certain rules. The advanced classes were considered to consist of a bright group of equals who caught on quickly to new concepts, therefore needing less time to be spent on discussion of concepts and possible strategies. Coupled with a concentration of work from the textbook, this encouraged more formal thinking about mathematics in the advanced classes than in the pilot class. Constructivist activities occurred most naturally in the heterogeneous pilot class, where there was more discussion of concepts and possible strategies for solving problems.

6. The construction of students' mathematical knowledge is dependent on the mathematical environment in which they are placed.

Opportunities to actively construct mathematical knowledge differed in the various classrooms because classes were exposed to different kinds of curriculum and instruction. The teacher found it difficult to follow his leanings toward constructivism in the basic classes where he was thrust into the role of sole instructional authority. The pilot and advanced classes received more opportunities to actively construct mathematical knowledge. These students experienced the same challenging mathematics curriculum and enjoyed using technology for problem solving.

Symbolic Interactionism and the Delivery of Instruction

The second major finding of this study suggests that labeling classes affects the teacher's approach to instruction. Symbolic interactionism holds that humans act towards things on the basis of the meaning that those things have for them. Because the teacher perceived the advanced and basic classes as homogeneous groups, this affected his instructional practices and the nature of mathematics that was available to students in these classes. Therefore, grouping students according to ability created unequal opportunities for the active construction of mathematical knowledge. The consonance of the findings of this study to the symbolic interactionist views of Berger and Luckmann (1967) on the social construction of reality adds strength to the study.

Five specific findings support the second major finding of the study:

1. The teacher as instructional leader was responsible for the enacted curriculum.

The teacher was responsible for what actually happened in the various classes and in these classes his teaching strategies differed. In the basic and advanced classes, students were perceived as homogeneous groups of students with similar needs. Although students' in basic classes mathematics scores ranged from the 5th to the 97th percentile, there was less of an adaptive response to the many levels of mathematical ability than in the pilot class.

2. The nature of direct teacher instruction varied between the different classes.

Although direct teacher instruction was the predominant teaching method, and students appreciated the clarity of the teacher's instruction, it differed in the various classes. In the basic classes the teacher asked questions that were straightforward and only required short answers. The usual pattern of interaction in the basic classes was 'question and answer'. Usually, in the advanced classes, the teacher demonstrated how to do the problems and then some students showed other methods. The teacher asked questions that required students to show how they found solutions. The usual pattern of interaction in the advanced classes was 'show and tell'. In the heterogeneous pilot class the direct teacher instruction time could best be described as 'interactive'. Students asked more questions of the teacher than students in other classes, and built upon the responses of other students.

3. In the traditional ability-grouped classes the teacher was more frequently thrust into a role of sole instructional authority.

Students in the advanced and basic classes relied more heavily on the teacher alone for help than students in the pilot class. High-achieving students in the basic classes were given the role of mentor but found it difficult to fulfill since the number of low-achievers at each table was too great. In the pilot class, students more frequently sought help from others at their table. Student-student mathematical interactions were commonplace.

4. Generally, students preferred other methods of mathematics instruction to using the textbook.

Students in the basic classes disliked their mathematics textbook; it was very similar to that of the previous year. While students in all classes had opportunities to use computers for practice exercises, the pilot and advanced classes were also exposed to interactive problem solving on the computers and on videos. They were challenged and motivated by these activities.

5. In the pilot class, students' individual mathematical differences were accommodated through instruction rather than through curriculum variations.

These students used more technology than students in basic classes, going to the computer room more frequently for work on basic skills, and using computers and videos for interactive problem solving.

Constructivists believe that learners construct their own mathematical knowledge, individually and cooperatively, from a tool kit of concepts, strategies and skills. It is the teacher's responsibility to establish mathematical environments where this can be achieved. Findings from the study indicated that opportunities for the construction of mathematical knowledge occurred most frequently in the pilot class, where students were exposed to a greater variety of mathematical situations. As they shared their personal mathematical experiences, students negotiated mathematical meaning through interactions with their peers. The emphasis on mathematical activity in a mathematical community is the common theme that runs through the teaching of constructivist mathematics teachers.

The Social Construction of Reality through Interactions

The third major finding of the study indicates that when heterogeneous and homogeneous mathematics classes were compared, different patterns of interaction emerged between the teacher and students, and between the students themselves. Constructivist theory emphasizes the need to establish mathematical communities where teachers are responsible for encouraging mathematical interactions between students, and for engaging in the kind of teacher-student interaction that promotes higher levels of thinking.

Six specific findings support the view that grouping arrangements determined the quality of social and mathematical interactions, and the reality of daily classroom life:

1. Low-achieving students in the basic classes were off-task for longer periods of time than low-achieving students in the pilot class.

Low-achieving students emulated the behavior of their peers. In the pilot class, low-achieving students had good role models and when they attempted to initiate inappropriate interactions they were usually ignored. As they worked in small groups, these students learned to appreciate the advantages of being on friendly terms with those with superior mathematical ability. They also received a more interesting curriculum than the students in basic classes and this helped to keep them on task. There were more negative student interactions in the basic classes, where the range of ability, from the 5th to the 97th percentile, resulted in a 1:4 ratio of high- to low-achievers. There were insufficient good role models in the basic classes and students were disadvantaged by having few good students with whom to have mathematical interactions. They were unmotivated by activities that consisted mainly of rote learning and computation, and became bored. This resulted in off-task, negative behaviors.

2. The composition of the pilot class resulted in more meaningful social and mathematical interactions taking place between students than in other classes.

Most of the mathematics was challenging and new to students in the pilot class; they endeavored to make sense of the instruction they received through questioning the teacher and through engaging in mathematical interactions with their peers. Students in the pilot class used each other as resources; the top students in the pilot classes gained self-assurance when they assumed responsibility for helping others. As they worked together students learned to trust each other and were less inhibited than those in the advanced classes about seeking advice from others.

3. Social and mathematical interactions were limited in the advanced classes.

Students in these classes came from similar backgrounds and they tended to be socially homogeneous. There were no Afro-American students in the advanced classes and students had fewer opportunities to interact with others from diverse backgrounds. Generally the advanced classes were competitive; they enjoyed interacting with the teacher but saw less reason to interact mathematically with their peers. In these classes there was more mathematical interaction between teacher and students than between students.

4. Teacher-student interaction was a product of the social organization of classrooms.

In the basic classes the teacher was thrust into a role of instructional authority that primarily consisted of controlling negative interactions as students worked on basic concepts. In order to maximize the learning time in the basic classes, the teacher had to tacitly condone many off-task behaviors. In the pilot and advanced classes the teacher acted more as a facilitator, and sought to encourage mathematical thinking. A special empathetic relationship existed between the teacher and students in the advanced classes, and the teacher frequently shared personal experiences and anecdotes with these students.

5. The heterogeneous pilot class best accommodated the mathematical and social needs of students from under-privileged backgrounds.

Positive social and mathematical interactions occurred more often in the pilot class than in the basic classes and this contributed to the learning process. Interactions that took place in the pilot class between students, and between the teacher and students, created a supportive environment. Because of the heterogeneity of the group, seating arrangements were frequently changed and students had many opportunities to make new friends. Social norms had to be renegotiated in the pilot class. In previous mathematics classes these students had been placed with others from similar backgrounds and with similar mathematical ability. Now they learned to interact with and appreciate the diversity of others.

6. Students in the various classes learned different social tools and were exposed to different social language and customs.

Students in the basic classes engaged in more inappropriate social behaviors; they called each other names, took other students' belongings, and were more physically interactive. There were fewer misbehaving students in the pilot class and more well-behaved students to act as role models; this reduced the number of inappropriate behaviors. Students in the advanced classes came from good homes. They were mostly polite, well-behaved, and interacted positively with their peers. There were no Afro-American students in these classes. Advanced students had fewer opportunities to interact with others from different backgrounds.

The reality of life in the various classrooms was the result of social and mathematical interactions that arose between students and between the teacher and students. The heterogeneous pilot class engaged in more teacher-student interaction for the purpose of understanding concepts, and indulged in more meaningful mathematical student-student talk than other classes. The challenging mathematics curriculum determined that students in the pilot class were compelled to interact with each other. Students needed to share mathematical resources and experiences. These students learned to appreciate the mathematical and social diversity of others.

This study focused primarily on mathematical and social interactions that take place in mathematics classes between students, and between the students and their teacher. This study is less concerned than others have been with the measurable mathematics achievement of students. The study is centered on the quality and meaning of mathematical activity. Qualitative methods of research were employed in this study. These consisted largely of observations conducted over the entire school year, and in-depth interviewing. Findings from this study generally support those of previous research.

Relationship of the Findings to Previous Research

Grouping students according to ability has long been a controversial issue in education and has resulted in hundreds of studies. Most of these studies are quantitative and are concerned with the effect on student achievement. In recent years researchers have used the technique of meta-analysis to perform exhaustive reviews of the research on ability grouping (Henderson, 1989; Kulik & Kulik, 1982, 1984; Noland & Taylor, 1986; Slavin, 1987, 1990). Conclusions indicated that no overall academic advantages were gained as a result of ability grouping, although some small gains were reported for students homogeneously grouped in special programs (Kulik, C. 1985). These meta-analyses of research on ability grouping also reported consistent negative effects on students' self-concept, especially for students placed in the lowest groups.

Findings from several other studies attributed students' under-achievement in mathematics to traditional methods of organizing students for mathematics according to their perceived mathematical ability (Eccles & Midgley, 1989; Stipek & MacIver, 1989; Useem, 1990; Usiskin, 1987). Other studies showed a steep decline in mathematical self-concept during early adolescence (Brassell, Petry, & Brooks, 1980; Eccles et al, 1983; Eccles & Midgley, 1989; Stipek & MacIver, 1989).

Recent studies have reported declining mathematical achievement in American schools and have documented the need for the constructivist perspective in mathematics (National Council of Teachers of Mathematics, 1989; National Research Council, 1989). Constructivist studies have stressed the importance of students constructing their own mathematical knowledge, individually and collaboratively, within a mathematical community (Maher & Davis, 1990; Lampert, 1990). Wood, Cobb and Yackel (1991) described how an elementary teacher changed her beliefs about mathematics instruction. The teacher came to realize that teaching and learning mathematics involved social interactions in which students are actively engaged in creating mathematical meaning.

Cobb, Yackel, and Wood (1992) suggested that while some constructivists have failed to emphasize the social and cultural nature of mathematics, others have diminished the importance of the role of the teacher.

The present study contains several important characteristics that relate its findings to, and complement, those of previous research. The first of these is that all students in the study had the same, very able, mathematics teacher. Other studies have shown that students in the low-ability grouped classes are frequently assigned less able teachers (Goodlad, 1984; Oakes, 1985, 1990). In this study, where the teacher's approach to instruction was a result of the social organization of classes, it was possible to describe the teacher's behaviors and perceptions as he interacted with the variously grouped classes.

Other researchers also reported that when students work in small groups, these groups are least effective when they are based on students' ability. They noted that cooperative learning activities were less likely to take place in ability grouped classes. Lack of exposure to small-group work may be one reason why high-ability students prefer individual work (Good et al., 1990; Janicki & Peterson, 1981).

Findings from the present study agree with those of other studies that found successful mathematics teachers to be active, well-organized, and strongly academic, emphasizing active whole-class instruction and devoting less time to seatwork. Effective teachers asked many high- and low-order questions (Evertson et al., 1980; Good, Grouws, Mason, Slavings, & Cramer, 1990). Findings from the present study suggest that teacher effectiveness results, in part, from the social organization of classes. Reality of classroom life was the result of interactions; these differed in the various classes. Interactions between the teacher and students were most empathetic in the advanced classes. On the other hand, mathematically meaningful interactions between students were most common in the pilot class.

Studies have shown that ability grouping limits interaction between students of various races and from different socioeconomic backgrounds (Damico, 1985; Metz, 1983; Oakes, 1985; Schofield, 1989). Findings from the present study supported those findings and found that students in the heterogeneous pilot class had the greatest number of opportunities to interact socially and mathematically with others from diverse backgrounds.

Studies have shown that traditional instructional strategies are resistant to change. Findings from this study agreed with other researchers who also found ability grouped classrooms to be traditional, teacher-directed environments (Goodlad, 1984; Oakes, 1985; Trimble, 1988). Oakes (1985, 1990) reported that students in low-ability grouped classes, usually the poor and minority students, receive an inferior education in terms of the content and quality of instruction and the quality of their school relationships.

In contrast to findings from Oakes' studies, where the lowest ability groups were often assigned the worst teachers, all students in the present study were exposed to the same very able teacher and the same pleasant physical environment. Yet other findings from the present study strongly support those of Oakes. There were more minority and poor students in the basic mathematics classes. There students received a boring, repetitive, curriculum, had less quality instructional time because of the number of negative student behaviors. Expectations were lower for these students and they were given less homework than students in other classes. Mathematics students in the basic classes were given few opportunities to attempt more challenging work. The findings show that the social organization of students determines the number and quality of learning opportunities in classrooms.

Implications

Findings from this study help determine the usefulness of the constructivist perspective in explaining differences in attitudes and achivement that result from variations in organizational and instructional practices.

Implications of the Findings for Researchers

The findings from this study will benefit researchers by providing insights that will help guide future research and educational practice. When researchers conduct studies that ask questions about the nature of mathematics, and what it means to engage in mathematical interactions, they should consider the advantages of constructivist methods of qualitative research. Such methods as those used in the present study may better answer these questions than traditional methods of educational research. In the present study qualitative methods of observation and in-depth interviewing permitted insights into how students construct mathematical knowledge in the various classes, and gauged the quality of their mathematical and social interactions.

The findings raise several issues that may have implications for other researchers as they seek to extend the knowledge base about how students learn mathematics in classes variously grouped for instruction. Findings from the study suggest that some grouping arrangements are less likely than others to induce students' engagement in meaningful mathematical activity. Other findings point to the possibility that more powerful mathematical constructions and more meaningful mathematical interactions take place when students work cooperatively in small, heterogeneous groups. As these findings are specific to this study, other researchers using similar qualitative methods may be directed to further research in an attempt to extend the generalizability of the findings. Would similar findings result from studies of students at other grade levels who are in classes variously grouped for instruction in mathematics? Is it possible for classes that are ability grouped for mathematics to engage successfully in constructivist activities? If not, what is it about the nature of social interactions of a heterogeneous group of students that encourages meaningful mathematical activity?

Recent reports have underscored the deplorable state of the mathematics proficiency of American students. Researchers have an important role to play in presenting evidence to educators, and the general populace, that traditional methods of organization and instruction in mathematics are unsuited for the skills that students will need in the twenty-first century.

Implications of the Findings for Practitioners

Ability grouping has long been considered the most effective way of organizing students for mathematics instruction. Findings from studies have demonstrated that teachers believe in the effectiveness of ability grouping and are resistant to change (Anderson & Barr, 1989). Studies have indicated that extremely heterogeneous classes call for teachers who are skilled classroom managers and who possess a wide variety of teaching strategies (Evertson, Sanford, & Emmer, 1981). Many other studies point out that ability-grouped classes tend to be bland, traditional, teacher-directed environments (Goodlad, 1984; Oakes, 1985; Trimble, 1988). 'Good' teachers are often considered to be those who maintain a quiet classroom, show students precisely how to do mathematics by providing a great deal of direct instruction, and who demand a great deal of textbook work.

Findings from the present study help to answer a question concerning the nature of mathematics instruction that was posed by John Goodlad (1984, p. 210). He asked why, when so many mathematics teachers stated that they wanted their students to have higher-order thinking skills, did so few teachers get beyond the kind of rote teaching and textbook dependency that was unlikely to develop powers of critical thinking?

In the present study the composition of classes determined the kind of mathematics taught and the nature of mathematical activities. The teacher responded to the cognitive demand of students in the heterogeneous pilot class for mathematical interaction and constructivist activities. Ability-grouped classes received more traditional kinds of mathematics instruction.

Findings from the study may be of interest to mathematics teachers, particularly those in middle schools. Teachers need to engage in reflective practices, individually and in mathematical discussions with their peers, on the nature of mathematics and appropriate learning activities. Researchers stress that mathematics teachers must reconsider the nature of mathematics, or what it means to think mathematically and to engage in mathematical activities (Davis, Maher & Noddings, 1990). How teachers think about mathematics has important implications for pedagogical practice.

Implications of the Findings for Educational Leaders

While the teacher as instructional leader is responsible for what happens in classrooms, the way students are placed in classes for instruction is the responsibility of administrators at the school and district level. Findings from this study indicate that the social organization of schools determines the reality of daily life in classrooms and the quality of social and mathematical interactions. Studies have indicated that vocal parents of high-ability children, who believe that ability grouping best fits the needs of their child, often influence grouping arrangements in schools (Oakes, 1985; 1990; Useem, 1990). It is therefore imperative that educational leaders educate parents concerning the advantages of mixed-ability classes where constructivist mathematical activities are more likely to take place. Educational leaders, parents, and teachers must engage in conversations about the nature of mathematics. Parents need to realize that constructivist methods of teaching mathematics are necessary for students who will reach adulthood in the twenty-first century.

Findings from the study pointed to the difficulty teachers have in changing their methods of instruction. Educational leaders must encourage and provide support for teachers seeking to change the nature of their mathematics classes, by providing resources and quality materials. In the present study successful alternative instructional strategies resulted from quality software, innovative videos, and problem solving that was specifically designed for cooperative groups. The use of simple and complex manipulatives should be a regular part of mathematics instruction. Teachers need workshops on how to provide and use constructivist methods and materials and on how to teach heterogeneous mathematics classes.

Educators of prospective mathematics teachers may find that some findings from this study benefit teacher preparation courses. Constructivism means that many traditional approaches to teacher education must be changed; student teachers must internalize the meaning of constructivism. Teacher educators need to model behaviors by reducing the amount of direct instruction and the number of lectures, and by providing videos of constructivist classrooms. Student teachers should work together in small groups on problem solving; they should strive to find various methods and solutions and should present these to the class. Methods of evaluation may include collaborative efforts, presentations, projects, interviews, and individual work. Findings from the study suggest that students benefit from various instructional strategies. Therefore, students teachers should go into schools with 'state-of-the-art' knowledge concerning interactive videos, computer software, and mathematical manipulatives.

Finally, there may be implications in the findings that are of interest to political leaders. Studies that project demographic changes predict that the number of poor and minority students in American schools will continue to grow rapidly. Education is the key to upward mobility for the under-privileged who are striving for improvement in their social and economic condition. An educated populace is essential for the future prosperity of this country and for the maintenance of a civilized, democratic society. The role of schools is to ensure that all students become productive members of society.

This study supports the constructivist view that the formation of academic ability is a social construction. The major finding from this study indicated that interactions that take place between students and the mathematical environment in which they are placed may be responsible for differences in students' mathematical achievement. The pilot class offered students of all abilities educational equality and equal opportunity. All students benefitted when individual differences in ability were accommodated through instruction rather than curriculum variations.

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APPENDIX

STUDENT INTERVIEW GUIDE

The interview will begin with some general questions designed to relax the interviewee. For example:

Do you like being in 6th grade?

What are your favorite classes?

What subject are you best at?

Questions will eventually focus on the following areas.

The social organization of the math class

What kind of math students are in this class?

How do you feel about being in this math class?

Do some students find math hard?

Do some students misbehave regularly in the classroom?

Do you like having a mix of good, average and poor students in the class? OR

Do you like having just good math students in the class?

Social relationships

Do you like working alone or do you like to work in a group?

What kinds of work do you do as a group?

If you could choose three people to work with to solve a problem who would you choose?

Have you made any new friends from the people in your math class?

Does it help the slower math students to be with people who are better than they are?

Instructional practices

What kinds of things do you do in this class?

Are these activities different from what you've done in other math classes?

How often do you work in small groups?

Do you like using computers?

How often do you use calculators during class?

What do you like best about math? Like worst?

How does the teacher help the (slower) students?

Do you think Mr. Abbot likes teaching math? What makes you think this?

Describe a typical day in your math class?

Attitude to math

Do you like coming to math class? Why/not?

Do you think that you doing well in math?

Do you get worried before a test?

What grade do you think you'll get in math?

What would you like to get?

Do you regularly do your homework?

How important do you think math is? Do you think you'll use this math in real life?

Give me some words to describe your math class?

Do you like working in small groups?

STUDENT ATTITUDE TO MATHEMATICS INVENTORY

Below are some sentences that describe how you might feel about mathematics. Please circle the letter

A	B	C	D	E
strongly agree	agree	disagree	strongly disgree	no opinion

1. I like math. A B C D E

2. The math textbook is too easy. A B C D E

3. Math isn't one of my favorite subjects.A B C D E

4. I like working in small groups during math class. A B C D E

5. I'd rather work a short easy problem than a long interesting one. A B C D E

6. My math teacher thinks math is a very important subject. A B C D E

7. I would like a job that doesn't use math. A B C D E

8. Math is easy for me. A B C D E

9. No matter how hard I try, I can't understand math. A B C D E

10. It's fun to work with math. A B C D E

11. People need math in most jobs. A B C D E

12. I think math is very boring. A B C D E

13. My math grades have usually been lower than my grades in other subjects. A B C D E

14. Math is very interesting to me. A B C D E

15. Word problems in math have always been difficult for me. A B C D E

16. You need math in order to get a good job. A B C D E

17. I feel tense and tight when someone talks about math. A B C D E

18. My math teacher makes math interesting. A B C D E

19. I think our math books are too hard. A B C D E

20. I'd like more small-group work in math. A B C D E

21. Math makes me worried and confused. A B C D E

22. The homework is usually interesting. A B C D E

23. I like working on my own better than working in a group.
A B C D E

24. My friends think math is important. A B C D E

25. My math teacher doesn't seem to enjoy teaching math.
A B C D E

26. Math is not a favorite subject of most of my friends.
A B C D E

27. We do a lot of small-group work in math class.
A B C D E

28. We use many materials other than the textbook in our math class. A B C D E

29. I feel relaxed and happy when doing math.
A B C D E

30. I don't like the math textbook. A B C D E

31. My math teacher is usually friendly and helpful.
A B C D E

32. I like to use the computer during math class.
A B C D E

33. Math is one of my favorite subjects. A B C D E

34. We have too much homework in math. A B C D E

35. Math is very important in everyday life. A B C D E

36. Most of the other students in my math class know more math than I do. A B C D E

37. I like math best when we don't use the textbook.
A B C D E

38. I worry about not doing well on math tests A B C D E

39. Math problems are usually about real-life A B C D E

40. I am doing worse in math this year than I did in other grades. A B C D E

QUESTIONS FOR STUDENTS

READ THE QUESTIONS CAREFULLY AND ANSWER WITH A FEW SENTENCES.

1. Which class do you look forward to most each day? What kinds of things happen in that class that you like?

2. What's the favorite thing that you like doing most in your math class?

3. What was the last new thing you learned in math? How did the teacher teach this to you?

4. Is it easier for you to work math problems on your own or when you work with a group? Why?

5. Describe what kind of math student you are.

6. Describe what usually happens during a math class.

7. Describe some of the different ways your class is taught math.

8. Explain the best way for you to learn math?

9. Have you made any new friends this year from people in your math class? How many? Describe how this new friend(s) is different from you?

CIRCLE AT LEAST THREE WORDS THAT DESCRIBE HOW YOU USUALLY FEEL ABOUT MATH CLASSES

worried easy interesting confusing hard fun

difficult happy boring challenging confident

CAN YOU THINK OF ANY OTHER WORDS TO DESCRIBE HOW YOU FEEL ABOUT MATH CLASSES?

BIOGRAPHICAL SKETCH

Jennifer S. Harrison was born in Edinburgh, Scotland and spent her early childhood in Norway. She was educated in Britain and graduated from the Girls' Grammar School in Colchester, Essex. She received her teaching certificate from the University of Birmingham, England, and taught for several years in the British school system before moving to the United States with her husband and three children.

Jennifer worked for Jefferson County Education Authority for five years, teaching mathematics in Evergreen, Colorado. In 1987 she received her Master's degree in education from the University of Colorado.

In 1987 she and her family relocated to Gainesville, Florida. For the last five years Jennifer has been teaching mathematics in Alachua County school district. She often acts as middle school mathematics consultant to schools and school districts, and has on several occasions acted as consultant for the Southeastern Desegregation Assistance Center on the subject of ability grouping.

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Paul S. George, Chair
Professor of Educational Leadership

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Joan L. Curcio

Joan L. Curcio
Associate Professor of Educational Leadership

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Sandra B. Damico

Sandra B. Damico
Professor of Foundations of Education

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Mary Grace Kantowski

Mary Grace Kantowski
Professor of Instruction and Curriculum

This dissertation was submitted to the Graduate Faculty of the College of Education and to the Graduate School and was accepted as partial fulfillment of the requirements for the degree of Doctor of Philosophy.

December, 1992

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